Mixed-Integer Nonlinear Optimization for District Heating Network Expansion

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Trier University

The Energy Transition

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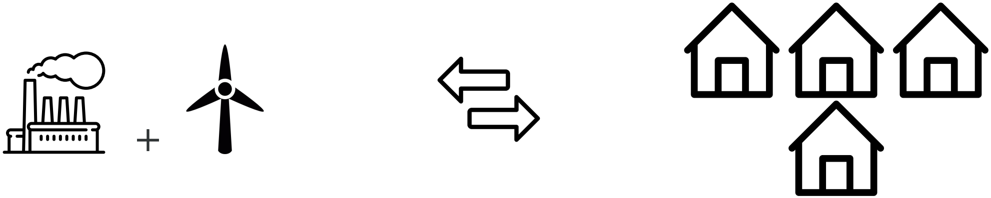

District Heating Networks and Optimization

Optimization:

- Control (Benonysson et al. 1995; Krug et al. 2020; Sandou et al. 2005; Verrilli et al. 2017)
- Design (Ameri and Besharati 2016; Blommaert et al. 2018; Bracco et al. 2013; Guelpa et al. 2018; Mertz et al. 2017; Söderman 2007)
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Challenges:

- Nonlinear behaviour of fluids in pipes
- Mixing of water at pipe intersections
- Water flow directions
- Modeling of **candidate** consumers

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Assumptions:

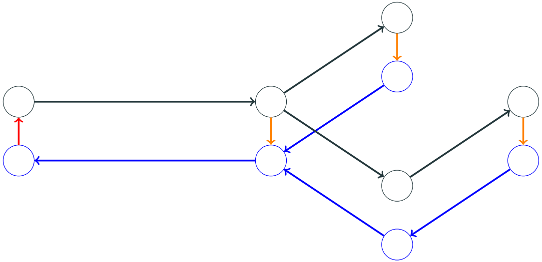
- One dimensional stationary model
- Only tree shaped networks

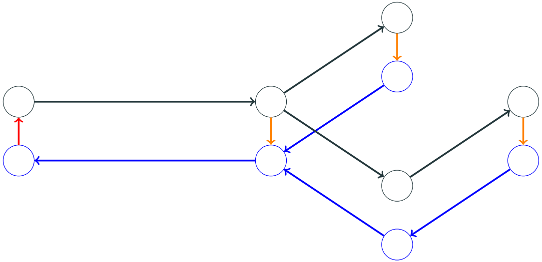
Overview

Expansion Problem

$$\begin{aligned} \max \quad & \text{Profit} = \text{Consumer Payment - Expansion, Operation and Maintenance Cost} \\ \text{s.t.} \quad & \text{System physics,} \\ & \text{Consumer constraints,} \\ & \text{Depot constraints.} \end{aligned}$$

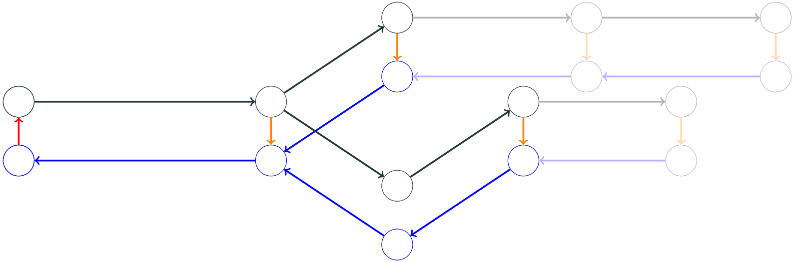
Network Modeling and Notation





We introduce the graph $G = (V, A)$ such that:

- A_{ff}, A_{bf}, A_{c}, a_{d}
- V_{ff}, V_{bf}



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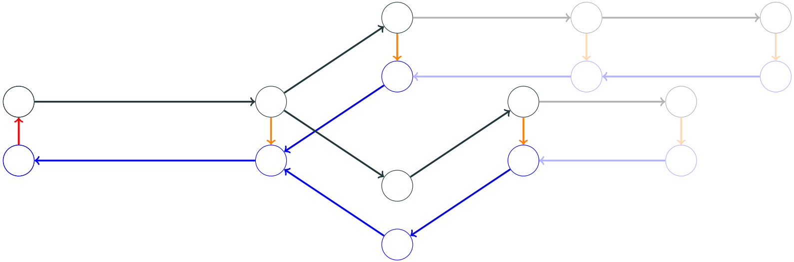
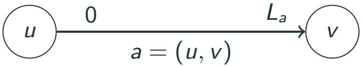


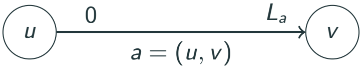
We introduce the graph $G = (V, A)$ such that:

- A_{ff}, A_{bf}, A_{c}, a_{d}
- V_{ff}, V_{bf}
- A^{e}_{ff}, A^{c}_{ff}, A^{e}_{bf}, A^{c}_{bf}, A^{e}_{c}, A^{c}_{c}
- V^{e}_{ff}, V^{c}_{ff}, V^{e}_{bf}, V^{c}_{bf}

Pipe Physics

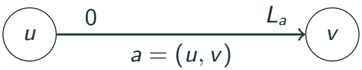


Pipe Physics



One-dimensional stationary momentum equation for compressible fluids in cylindrical pipes (Hauschild et al. 2020; Krug et al. 2020):

$$\frac{p_a(L_a) - p_a(0)}{L_a} = -g\rho h'_a - \lambda_a \frac{|v_a| v_a \rho}{2D_a}, \quad \forall a \in A_{\text{ff}} \cup A_{\text{bf}},$$



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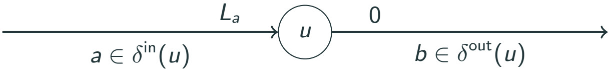
One-dimensional stationary thermal energy equation:

$$T_a(L_a; v_a) = \begin{cases} T_{\mathrm{soil}}, & v_a = 0 \\ (T_a(0) - T_{\mathrm{soil}})\, e^{-\frac{4U_a L_a}{c_p \rho D_a v_a}} + T_{\mathrm{soil}}, & v_a > 0 \end{cases}, \quad \forall a \in A_{\mathrm{ff}} \cup A_{\mathrm{bf}}.$$

Node Physics



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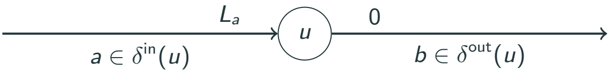
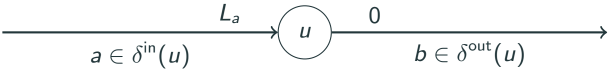


Mass flow continuity:

$$\sum_{a \in \delta^{\text{in}}(u)} q_a = \sum_{a \in \delta^{\text{out}}(u)} q_a, \quad u \in V.$$

Node Physics



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Pressure continuity:

$$p_u = p_a(0), \quad u \in V, \ a \in \delta^{\text{out}}(u),$$
$$p_u = p_a(L_a), \quad u \in V, \ a \in \delta^{\text{in}}(u).$$

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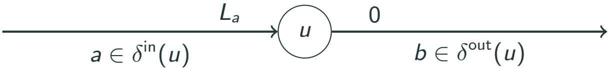

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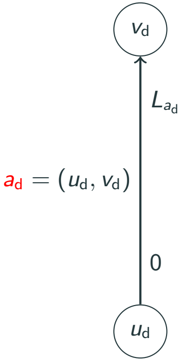
Temperature mixing (Krug et al. 2020):

$$T_u = \frac{\sum_{a \in \delta^{\text{in}}(u)} c_{\mathrm{p}} q_a T_a(L_a)}{\sum_{a \in \delta^{\text{in}}(u)} c_{\mathrm{p}} q_a}, \quad u \in V,$$
$$T_u = T_a(0), \quad u \in V,\ a \in \delta^{\text{out}}(u).$$

Depot

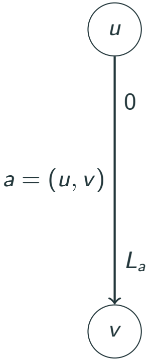
For $a_\text{d} = (u_\text{d}, v_\text{d})$:

$$
\begin{aligned}
p_{u_\text{d}} &= p_\text{s}\,, \\
P_\text{p} &= \frac{q_{a_\text{d}}}{\rho}\left(p_{v_\text{d}} - p_{u_\text{d}}\right), \\
P_\text{w} + P_\text{g} &= q_{a_\text{d}} c_\text{p} \left(T_a(L_{a_\text{d}}) - T_a(0)\right).
\end{aligned}
$$



Consumers

$$
\begin{aligned}
P_a &= q_a c_\mathrm{p} \left(T_a(0) - T_a(L_a) \right), && a \in A_\mathrm{c}^\mathrm{e}, \\
x_a P_a &= q_a c_\mathrm{p} \left(T_a(0) - T_a(L_a) \right), && a \in A_\mathrm{c}^\mathrm{c}, \\
T_a(L_a) &= T^\mathrm{bf}, && a \in A_\mathrm{c}^\mathrm{e} \cup A_\mathrm{c}^\mathrm{c}, \\
T_a(0) &\geq T_a^\mathrm{ff}, && a \in A_\mathrm{c}^\mathrm{e} \cup A_\mathrm{c}^\mathrm{c}, \\
p_v &\leq p_u, && a \in A_\mathrm{c}^\mathrm{e} \cup A_\mathrm{c}^\mathrm{c}.
\end{aligned}
$$



Thermal Energy Equation Approximation I

$$f(v_a, T_a(0), T_a(L_a)) := \begin{cases} \left(T_a(0) - T_{\text{soil}}\right) e^{-\frac{4U_a L_a}{c_{\mathrm{p}} \rho D_a v_a}} + T_{\text{soil}} - T_a(L_a), & v_a > 0, \\ T_{\text{soil}} - T_a(L_a), & v_a = 0. \end{cases}$$

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$$\Downarrow$$

$$f_{\text{approx}}(v_a, T_a(0), T_a(L_a)) := \sum_{(k,l,m) \in \Theta_d} \alpha_{klm} \, v_a^k \, T_a(0)^l \, T_a(L_a)^m + T_a(L_a) - T_{\text{soil}},$$

with

$$\Theta_d := \left\{ (k, l, m) \in \mathbb{N}^3 \colon k \neq 0 \text{ and } k + l + m \leq d \right\}.$$

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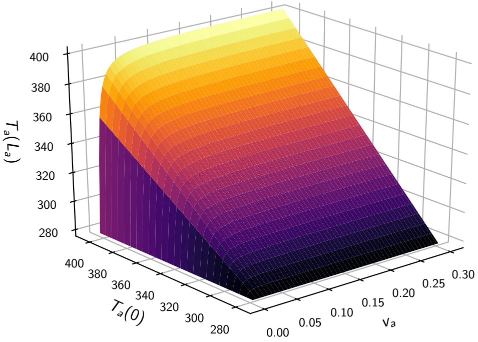
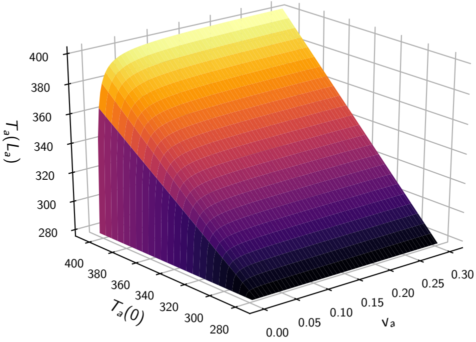
$$\Downarrow$$

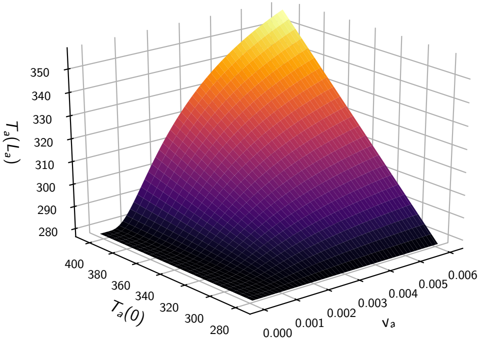
$$\min_{\alpha} \quad \sum_{i \in I} f_{\text{approx}}(v_a^i, T_a(0)^i, T_a(L_a)^i)^2.$$

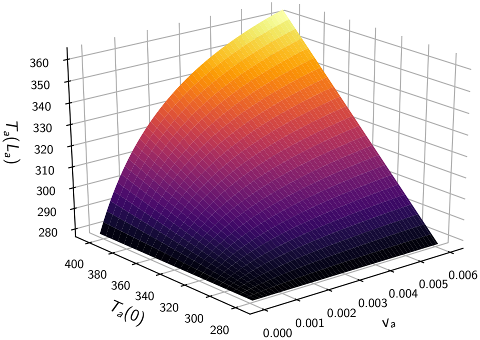
Thermal Energy Equation Approximation II



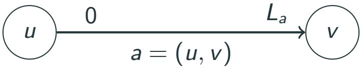


Thermal Energy Equation Approximation III





Candidate Pipe Modeling: Pressure Constraints



One-dimensional stationary momentum equation for compressible fluids in cylindrical pipes:

$$p_v - p_u + L_a g \rho h'_a + \lambda_a \frac{|v_a| v_a \rho L_a}{2D_a} = 0, \quad \forall a \in A_{\text{ff}} \cup A_{\text{bf}}.$$

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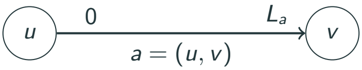

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And,

$$p_v - p_u + L_a g \rho h'_a + \lambda_a \frac{|v_a| v_a \rho L_a}{2D_a} \leq (1 - x_a) M^1_a, \quad \forall a = (u, v) \in A^{\mathrm{c}}_{\mathrm{ff}} \cup A^{\mathrm{c}}_{\mathrm{bf}},$$

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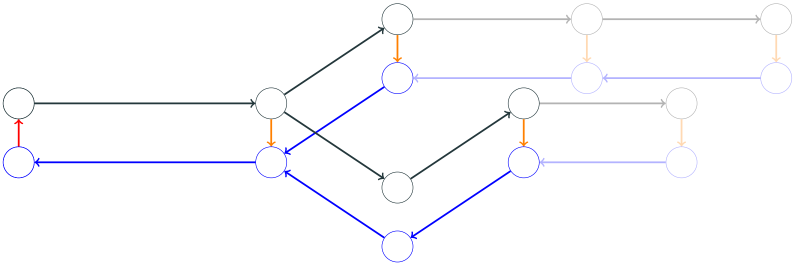
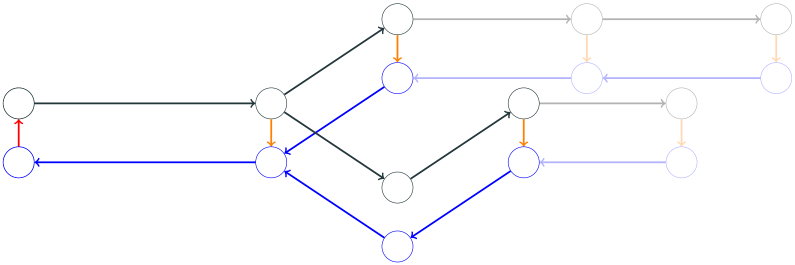
Candidate Pipe Modeling: Valid Inequalities





Valid inequalities:

$$x_{a_1} \leq x_{a_2}, \quad a_1, a_2 \in A_{\mathrm{ff}}^{\mathrm{c}} \cup A_{\mathrm{bf}}^{\mathrm{c}} \cup A_{\mathrm{c}}^{\mathrm{c}}, a_2 \in P(a_1).$$

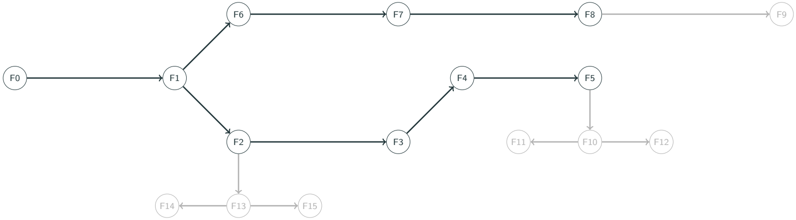
Objective

$$\max \quad \sum_{a \in A^{c}_{c}} P_a w \pi x_a - \sum_{a \in A^{c}_{ff} \cup A^{c}_{bf} \cup A^{c}_{c}} C^{\text{inv}}_a x_a - w \left(C_{\mathrm{p}} P_{\mathrm{p}} + C_{\mathrm{w}} P_{\mathrm{w}} + C_{\mathrm{g}} P_{\mathrm{g}} \right)$$

Model Summary

$$
\begin{aligned}
\max \quad & \text{objective}, \\
\text{s.t.} \quad & \text{stationary incompressible Euler equation}, \\
& \text{stationary thermal energy equation approximation}, \\
& \text{mass conservation}, \\
& \text{pressure continuity}, \\
& \text{temperature mixing equations}, \\
& \text{depot constraints}, \\
& \text{consumer constraints}, \\
& \text{system bounds}, \\
& \text{power bounds}, \\
& \text{valid binary inequalities}.
\end{aligned}
$$

Test Case



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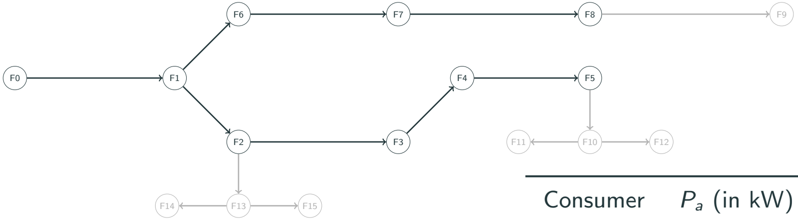

Cost	C_w	C_g	C_p
Value (€/kWh)	0	0.0415	0.165

Edge	$a \in A_c^c$	$a \in A_{ff}^c \cup A_{bf}^c$
C_a^{inv} (€)	100 000	[90 000, 330 000]

Consumer	P_a (in kW)
(F2,B2)	200.00
(F3,B3)	600.00
(F5,B5)	150.00
(F6,B6)	666.66
(F8,B8)	200.00
(F9,B9)	183.33
(F11,B11)	183.33
(F12,B12)	183.33
(F14,B14)	183.33
(F15,B15)	183.33

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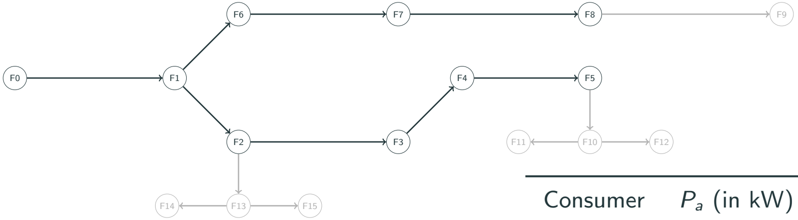

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Intel(R) Core(TM) i7-8550U, 16 GB RAM.
ANTIGONE using the Pyomo interface.

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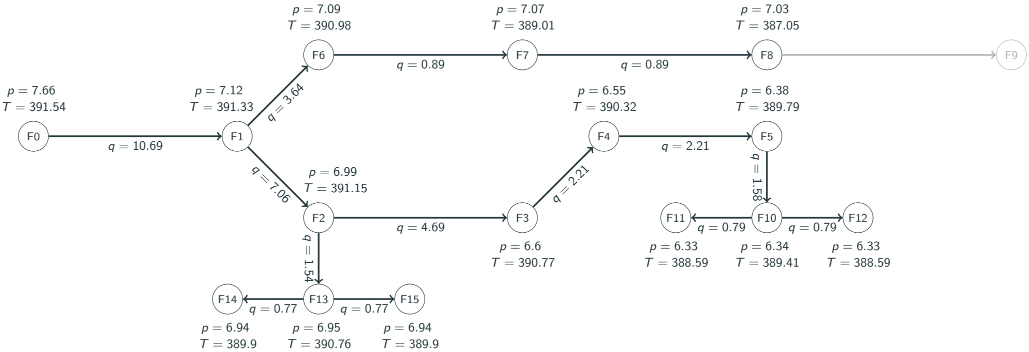
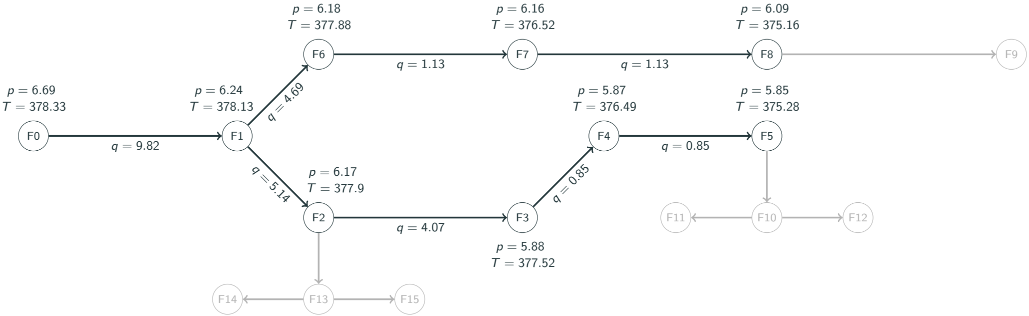
Initial Results



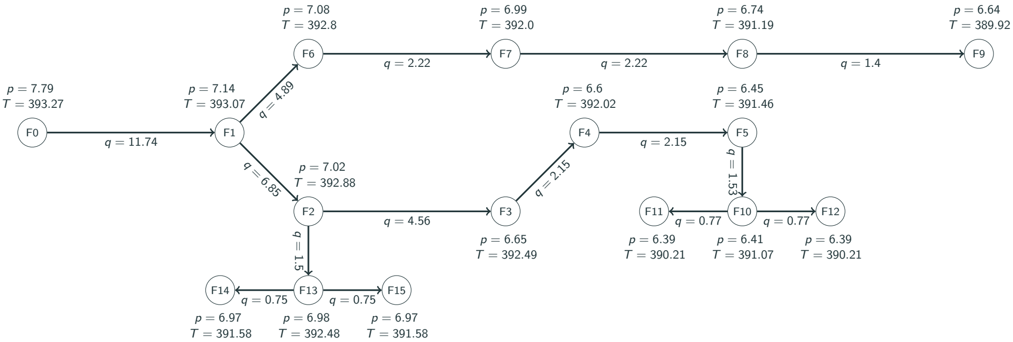
Discussion: Impact of Estimated Average Demand

If $P_a = 150\,\mathrm{kW}$ for all $a \in A_c^c$:



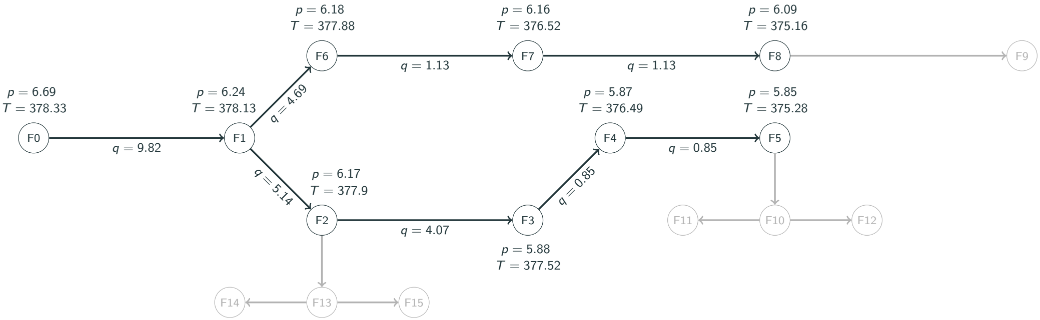
Discussion: Impact of Distance

If $P_a = 350\,\mathrm{kW}$ for all $a \in A_c^c$:



Discussion: Impact of Thermal Losses

If $U_a = 0.5\,\mathrm{W\,m^{-1}\,K^{-2}} \Rightarrow 0.8\,\mathrm{W\,m^{-1}\,K^{-2}}$:



Conclusion

Contributions of the expansion model:

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- Nonlinear modeling of T and p behavior

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- Polynomial approximation of the Thermal Energy Equation

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Main parameters:

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- Distance of the candidate consumer

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Contributions of the expansion model:

- Nonlinear modeling of T and p behavior
- Polynomial approximation of the Thermal Energy Equation

Main parameters:

- Estimated average demand
- Distance of the candidate consumer
- Pipe insulation