Mixed-Integer Nonlinear Optimization for District Heating Network Expansion

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14 September 2020, YMSGR 2020

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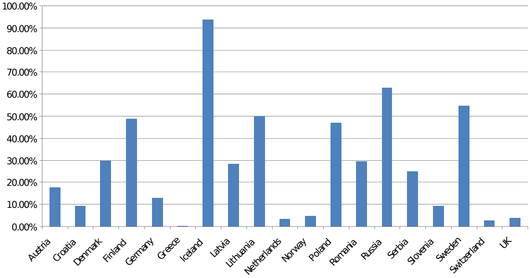
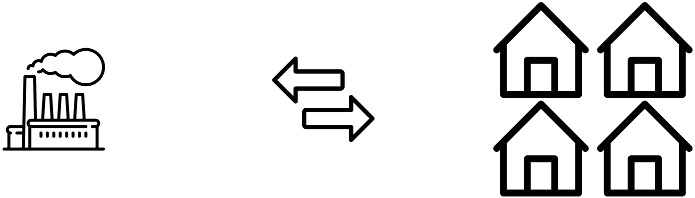
"Heating and cooling in buildings and industry accounts for **half** of the EU's energy consumption."

- Use cleaner energy
- Distribute energy in an efficient way

$\Rightarrow$ District Heating Networks



District Heating Systems and Optimization



Optimization:

- Control
- Design
- Expansion

Overview

Expansion Problem

$$\begin{aligned} \max \quad & \text{Profit} = \text{Consumer Payment - Expansion, Operation and Maintenance Cost} \\ \text{s.t.} \quad & \text{System physics,} \\ & \text{Consumer constraints,} \\ & \text{Depot constraints.} \end{aligned}$$

Challenges and Assumptions

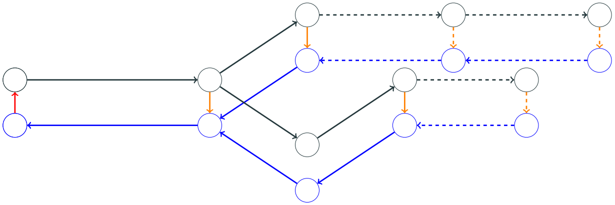
Challenges:

- Nonlinear behaviour of fluids in pipes
- Mixing of water at pipe intersections
- Water flow directions
- Modeling of **candidate** consumers

Assumptions:

- One dimensional stationary model
- Only tree shaped networks

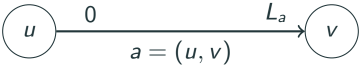
Network Modeling and Notation



We introduce the graph $G = (V, A)$ such that:

- $A_{\mathrm{ff}}, A_{\mathrm{bf}}, A_{\mathrm{c}}, a_{\mathrm{d}}$
- $V_{\mathrm{ff}}, V_{\mathrm{bf}}$
- $A^{\mathrm{e}}_{\mathrm{ff}}, A^{\mathrm{c}}_{\mathrm{ff}}, A^{\mathrm{e}}_{\mathrm{bf}}, A^{\mathrm{c}}_{\mathrm{bf}}, A^{\mathrm{e}}_{\mathrm{c}}, A^{\mathrm{c}}_{\mathrm{c}}$
- $V^{\mathrm{e}}_{\mathrm{ff}}, V^{\mathrm{c}}_{\mathrm{ff}}, V^{\mathrm{e}}_{\mathrm{bf}}, V^{\mathrm{c}}_{\mathrm{bf}}$

Pipe Physics



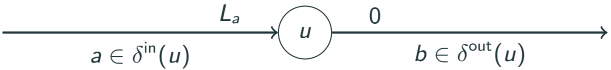
One-dimensional stationary momentum equation for compressible fluids in cylindrical pipes:

$$\frac{p_a(L_a) - p_a(0)}{L_a} = -g\rho h_a' - \lambda_a \frac{|v_a| v_a \rho}{2D_a},$$

One-dimensional stationary thermal energy equation:

$$T_a(L_a; v_a) = \begin{cases} T_{\text{soil}}, & v_a = 0, \\ (T_a(0) - T_{\text{soil}})\, e^{-\frac{4U_a L_a}{c_{\mathrm{p}}\rho D_a v_a}} + T_{\text{soil}}, & v_a > 0. \end{cases}$$

Node Physics



Mass flow continuity:

$$\sum_{a \in \delta^{\text{in}}(u)} q_a = \sum_{a \in \delta^{\text{out}}(u)} q_a, \quad u \in V.$$

Pressure continuity:

$$p_u = p_a(0), \quad u \in V,\ a \in \delta^{\text{out}}(u),$$
$$p_u = p_a(L_a), \quad u \in V,\ a \in \delta^{\text{in}}(u).$$

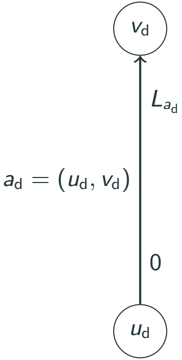
Temperature mixing:

$$T_u = \frac{\sum_{a \in \delta^{\text{in}}(u)} c_{\mathrm{p}} q_a T_a(L_a)}{\sum_{a \in \delta^{\text{in}}(u)} c_{\mathrm{p}} q_a}, \quad u \in V,$$
$$T_u = T_a(0), \quad u \in V,\ a \in \delta^{\text{out}}(u).$$

Depot

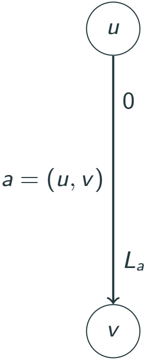
For $a_\mathrm{d} = (u_\mathrm{d}, v_\mathrm{d})$:

$$
\begin{aligned}
p_{u_\mathrm{d}} &= p_\mathrm{s}, \\
P_\mathrm{p} &= \frac{q_{a_\mathrm{d}}}{\rho} \left(p_{v_\mathrm{d}} - p_{u_\mathrm{d}} \right), \\
P_\mathrm{w} + P_\mathrm{g} &= q_{a_\mathrm{d}} c_\mathrm{p} \left(T_a(L_{a_\mathrm{d}}) - T_a(0) \right).
\end{aligned}
$$



Consumers

$$
\begin{aligned}
P_a &= q_a c_\mathrm{p} \left(T_a(0) - T_a(L_a) \right), && a \in A_\mathrm{c}^\mathrm{e}, \\
x_a P_a &= q_a c_\mathrm{p} \left(T_a(0) - T_a(L_a) \right), && a \in A_\mathrm{c}^\mathrm{c}, \\
T_a(L_a) &= T^\mathrm{bf}, && a \in A_\mathrm{c}, \\
T_a(0) &\geq T_a^\mathrm{ff}, && a \in A_\mathrm{c}, \\
p_v &\leq p_u, && a \in A_\mathrm{c}.
\end{aligned}
$$



Thermal Energy Equation Approximation I

$$f(v_a, T_a(0), T_a(L_a)) := \begin{cases} \left(T_a(0) - T_{\text{soil}}\right) e^{-\frac{4U_a L_a}{c_{\mathrm{p}} \rho D_a v_a}} + T_{\text{soil}} - T_a(L_a), & v_a > 0, \\ T_{\text{soil}} - T_a(L_a), & v_a = 0. \end{cases}$$

$$\Downarrow$$

$$f_{\text{approx}}(v_a, T_a(0), T_a(L_a)) := \sum_{(k,l,m) \in \Theta_d} \alpha_{klm} \, v_a^k \, T_a(0)^l \, T_a(L_a)^m + T_a(L_a) - T_{\text{soil}},$$

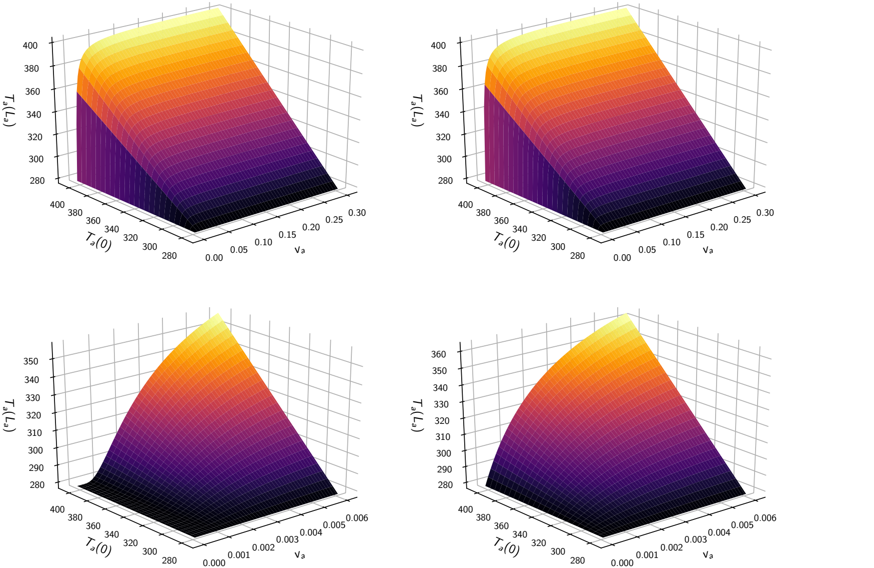
with

$$\Theta_d := \left\{ (k, l, m) \in \mathbb{N}^3 \colon k \neq 0 \text{ and } k + l + m \leq d \right\}.$$

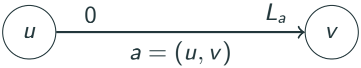
$$\Downarrow$$

$$\min_{\alpha} \quad \sum_{i \in I} f_{\text{approx}}(v_a^i, T_a(0)^i, T_a(L_a)^i)^2.$$

Thermal Energy Equation Approximation II



Candidate Pipe Modeling: Pressure Constraints



One-dimensional stationary momentum equation for compressible fluids in cylindrical pipes:

$$p_v - p_u + L_a g \rho h_a' + \lambda_a \frac{|v_a| v_a \rho L_a}{2 D_a} = 0,$$

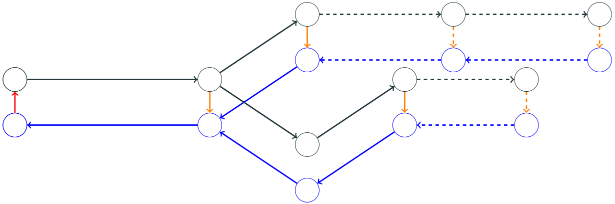
$$\Downarrow$$

For $a = (u, v) \in A_{\text{ff}}^{\text{c}} \cup A_{\text{bf}}^{\text{c}}$:

$$p_v - p_u + L_a g \rho h_a' + \lambda_a \frac{|v_a| v_a \rho L_a}{2 D_a} \leq (1 - x_a) M_a^1,$$

$$p_v - p_u + L_a g \rho h_a' + \lambda_a \frac{|v_a| v_a \rho L_a}{2 D_a} \geq -(1 - x_a) M_a^2.$$

Candidate Pipe Modeling: Valid Inequalities



Valid additional constraints:

$$x_a \leq x_{\bar{a}}, \quad \bar{a} \in P(a).$$

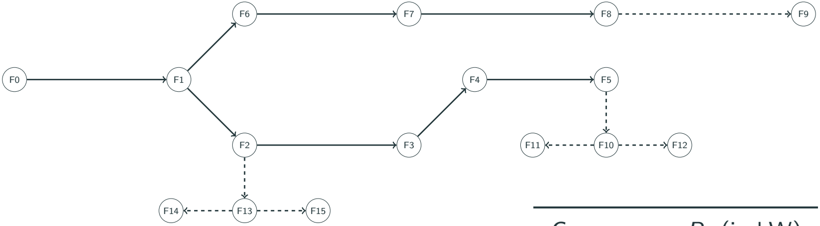
Objective

$$\max \quad \sum_{a \in A^{\mathsf{c}}_{\mathsf{c}}} P_a w \pi x_a - \sum_{a \in A^{\mathsf{c}}} C_a^{\mathsf{inv}} x_a - w \left(C_{\mathsf{p}} P_{\mathrm{p}} + C_{\mathsf{w}} P_{\mathrm{w}} + C_{\mathsf{g}} P_{\mathrm{g}} \right)$$

Model Summary

$$
\begin{aligned}
\max \quad & \text{objective}, \\
\text{s.t.} \quad & \text{stationary incompressible Euler equation}, \\
& \text{stationary thermal energy equation approximation}, \\
& \text{mass conservation}, \\
& \text{pressure continuity}, \\
& \text{temperature mixing equations}, \\
& \text{depot constraints}, \\
& \text{consumer constraints}, \\
& \text{system bounds}, \\
& \text{power bounds}, \\
& \text{valid binary inequalities}.
\end{aligned}
$$

Test Case



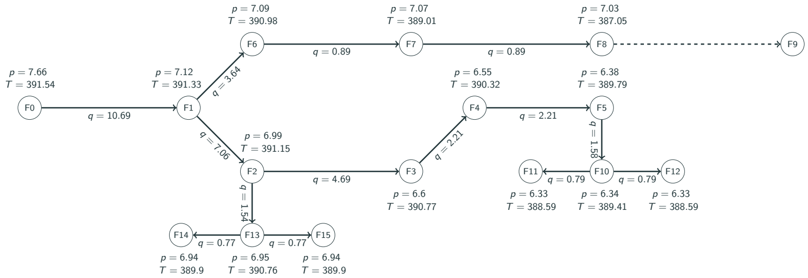
Cost	C_w	C_g	C_p
Value (€/kWh)	0	0.0415	0.165

Edge	$a \in A_\mathrm{c}^\mathrm{c}$	$a \in A_\mathrm{ff}^\mathrm{c} \cup A_\mathrm{bf}^\mathrm{c}$
C_a^inv (€)	100 000	[90 000, 330 000]

Consumer	P_a (in kW)
(F2,B2)	200.00
(F3,B3)	600.00
(F5,B5)	150.00
(F6,B6)	666.66
(F8,B8)	200.00
(F9,B9)	183.33
(F11,B11)	183.33
(F12,B12)	183.33
(F14,B14)	183.33
(F15,B15)	183.33

Intel(R) Core(TM) i7-8550U, 16 GB RAM.
ANTIGONE using the Pyomo interface.

Initial Results

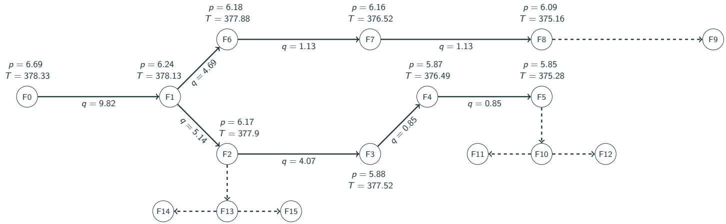


Objective (€/day)	P_{t} (kW)	P_{p} (kW)	P_{w} (kW)	P_{g} (kW)	P_{tl} (kW)
−1380.02	2550.0	2.84	500.0	2149.87	99.87

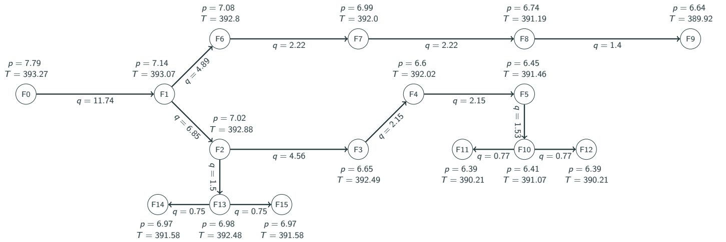
$$P_{\mathrm{t}} := \sum_{a \in A_{\mathrm{c}}^{\mathrm{e}}} P_a + \sum_{a \in A_{\mathrm{c}}^{\mathrm{c}}} x_a P_a, \qquad P_{\mathrm{tl}} := P_{\mathrm{w}} + P_{\mathrm{g}} - P_{\mathrm{t}}.$$

Discussion: Impact of Estimated Average Demand and Distance

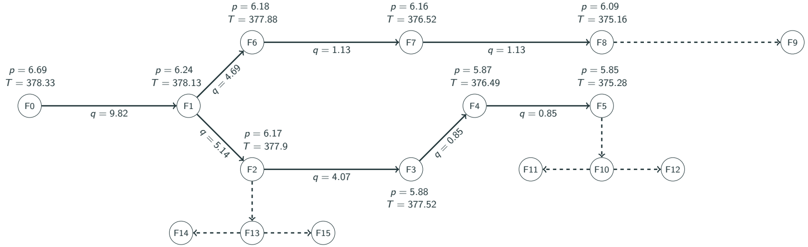
If $P_a = 150\,\text{kW}$ for all $a \in A_c^c$:



If $P_a = 350\,\text{kW}$ for all $a \in A_c^c$:



If $U_a = 0.5\,\mathrm{W\,m^{-1}\,K^{-2}} \Rightarrow 0.8\,\mathrm{W\,m^{-1}\,K^{-2}}$:



Conclusion

Contributions of the expansion model:

- Nonlinear modeling of T and p behavior
- Polynomial approximation of the Thermal Energy Equation

Main parameters:

- Estimated average demand
- Distance of the candidate consumer
- Pipe insulation

Some References

- Chiara Bordin, Angelo Gordini, and Daniele Vigo. "An optimization approach for district heating strategic network design." In: *European Journal of Operational Research* 252.1 (2016), pp. 296–307.
- Richard Krug, Volker Mehrmann, and Martin Schmidt. "Nonlinear Optimization of District Heating Networks." In: *arXiv preprint arXiv:1910.06453* (2019).
- Marius Roland and Martin Schmidt. "Mixed-Integer Nonlinear Optimization for District Heating Network Expansion." In: *Optimization Online* (2020).