Multidimensional Disjunctive Inequalities for Linear Chance-Constrained Problems with Finite Support

Discrete Optimization Talks

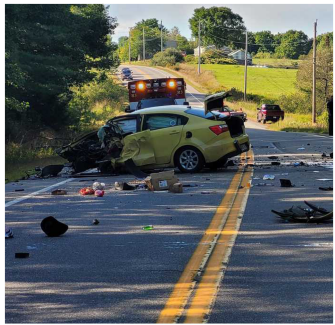
Diego Cattaruzza, Martine Labbé, Matteo Petris, Marius Roland, Martin Schmidt

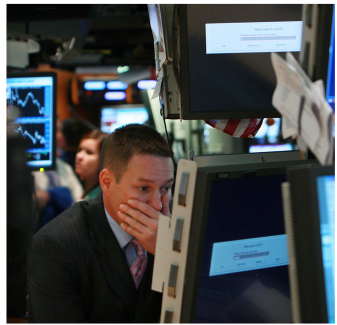
INRIA Lille, France

April 24, 2026

How much attention do extreme events deserve?







Chance Constrained Stochastic Programs (CCSPs)

$$
\begin{aligned}
v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\
\text{s.t.} \quad & x \in X(\xi)
\end{aligned}
$$

Chance Constrained Stochastic Programs (CCSPs)

$$
\begin{aligned}
v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\
\text{s.t.} \quad & \mathbb{P}_\xi[x \in X(\xi)] = 1
\end{aligned}
$$

Chance Constrained Stochastic Programs (CCSPs)

$$
\begin{aligned}
v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\
\text{s.t.} \quad & \mathbb{P}_\xi[x \in X(\xi)] \geq 1 - \tau
\end{aligned}
$$

Overview

1. Introduction
2. Setting and Challenges
3. Multi Dimensional Disjunctive Inequalities
4. Numerical Experiments
5. Conclusion

Setting and Challenges

CCSP with Finite Support

Assume

$$\xi \in \{\xi^s \in \mathbb{R}^d : s \in S\}$$

CCSP with Finite Support

Assume

$$\xi \in \{\xi^s \in \mathbb{R}^d : s \in S\}$$

Parameters & Variables

$$p_s \in [0,1],\ X^s = X(\xi^s) \qquad z_s \in \{0,1\}$$

CCSP with Finite Support

Assume

$$\xi \in \{\xi^s \in \mathbb{R}^d : s \in S\}$$

Parameters & Variables

$$p_s \in [0,1],\ X^s = X(\xi^s) \qquad\qquad z_s \in \{0,1\}$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & \mathbb{P}_\xi[x \in X(\xi)] \geq 1 - \tau \end{aligned} \qquad \rightarrow \qquad \begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in S \\ & \sum_{s \in S} p_s z_s \geq 1 - \tau \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned}$$

CCSP with Finite Support

Assume

$$\xi \in \{\xi^s \in \mathbb{R}^d : s \in S\}$$

Parameters & Variables

$$p_s \in [0,1],\ X^s = X(\xi^s) \qquad\qquad z_s \in \{0,1\}$$

Model

$$v^* = \min_{x \in \mathcal{X}} \quad f(x) \qquad \rightarrow \qquad v^* = \min_{x \in \mathcal{X}} \quad f(x)$$

$$\text{s.t.} \quad \mathbb{P}_\xi[x \in X(\xi)] \geq 1 - \tau \qquad\qquad \text{s.t.} \quad \boxed{z_s = \mathbb{1}(x \in X^s), \quad s \in S}$$

$$\sum_{s \in S} p_s z_s \geq 1 - \tau$$

$$z_s \in \{0,1\}, \quad s \in S$$

CCSP with Finite Support

Assume

$$\xi \in \{\xi^s \in \mathbb{R}^d : s \in S\}$$

Parameters & Variables

$$p_s \in [0,1],\ X^s = X(\xi^s) \qquad\qquad z_s \in \{0,1\}$$

Model

$$v^* = \min_{x \in \mathcal{X}} \quad f(x) \qquad \rightarrow \qquad v^* = \min_{x \in \mathcal{X}} \quad f(x)$$

$$\text{s.t.} \quad \mathbb{P}_\xi[x \in X(\xi)] \geq 1 - \tau \qquad\qquad \text{s.t.} \quad z_s = \mathbb{1}(x \in X^s), \quad s \in S$$

$$\sum_{s \in S} p_s z_s \geq 1 - \tau$$

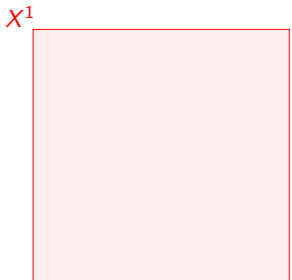
$$z_s \in \{0,1\}, \quad s \in S$$

Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} p_s z_s \geq 1 - \tau \\
& z_s \in \{0, 1\}
\end{aligned}
$$

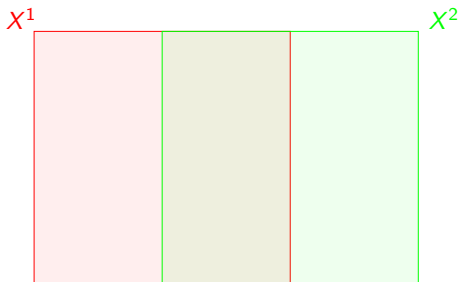
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} p_s z_s \geq 1 - \tau \\
& z_s \in \{0, 1\}
\end{aligned}
$$



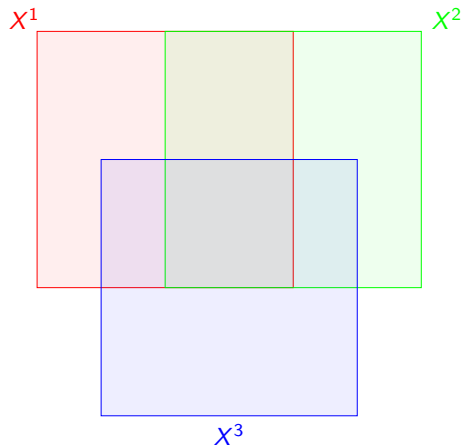
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} p_s z_s \geq 1 - \tau \\
& z_s \in \{0, 1\}
\end{aligned}
$$



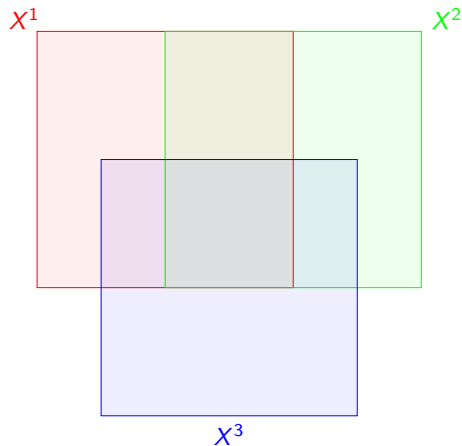
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} p_s z_s \geq 1 - \tau \\
& z_s \in \{0, 1\}
\end{aligned}
$$



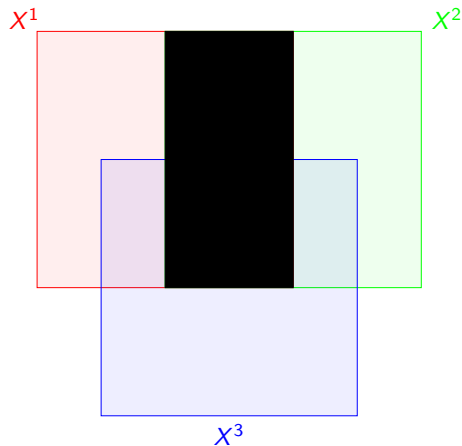
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



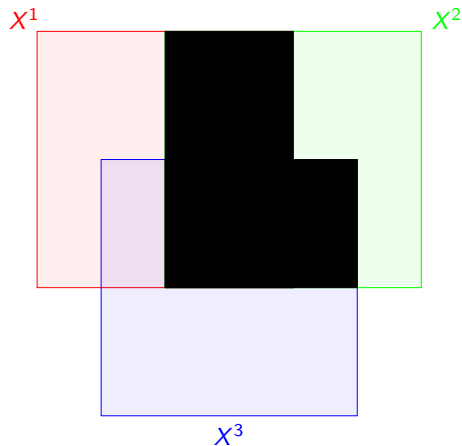
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



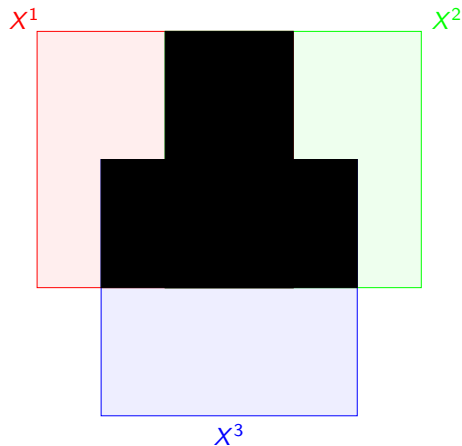
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



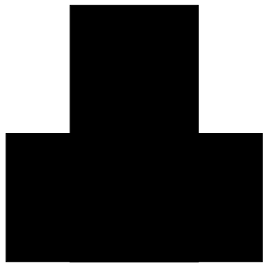
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



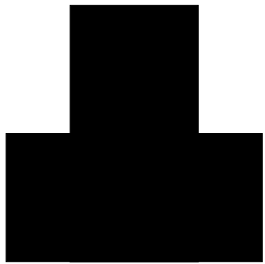
Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



Challenge 1: Nonconvexity & Nonsmoothness

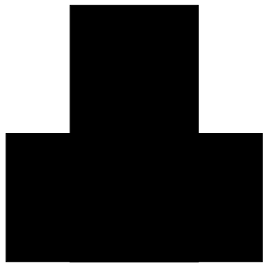
$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



$\rightarrow$ Noncovex nonsmooth x-space and $\binom{|S|}{\lfloor \tau |S| \rfloor}$ operations

Challenge 1: Nonconvexity & Nonsmoothness

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & f(x) \\
\text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in \{1, 2, 3\} \\
& \sum_{s \in \{1,2,3\}} z_s \geq 2 \\
& z_s \in \{0, 1\}
\end{aligned}
$$



→ Noncovex nonsmooth x-space and $\binom{|S|}{\lfloor \tau |S| \rfloor}$ operations

→ NP-hard to solve even for simplified cases (Luedtke et al. 2010)

Linear CCSP with Equiprobable Scenarios

Assumptions

$$p_s = \frac{1}{|S|} \qquad X^s = \{x \in \mathbb{R}^n : \ A^s x \geq b^s\} \qquad f(x) = c^\top x$$

Linear CCSP with Equiprobable Scenarios

Assumptions

$$p_s = \frac{1}{|S|} \qquad X^s = \{x \in \mathbb{R}^n : \ A^s x \geq b^s\} \qquad f(x) = c^\top x$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in S \\ & \sum_{s \in S} p_s z_s \geq 1 - \tau \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned} \qquad \rightarrow \qquad \begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & z_s = \mathbb{1}\big(A^s x \geq b^s\big), \quad s \in S \\ & \sum_{s \in S} z_s \geq |S| - \ \tau |S| \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned}$$

Linear CCSP with Equiprobable Scenarios

Assumptions

$$p_s = \frac{1}{|S|} \qquad X^s = \{x \in \mathbb{R}^n : \ A^s x \geq b^s\} \qquad f(x) = c^\top x$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in S \\ & \sum_{s \in S} p_s z_s \geq 1 - \tau \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned} \qquad \rightarrow \qquad \begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & z_s = \mathbb{1}(A^s x \geq b^s), \quad s \in S \\ & \sum_{s \in S} z_s \geq |S| - \tau |S| \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned}$$

Linear CCSP with Equiprobable Scenarios

Assumptions

$$p_s = \frac{1}{|S|} \qquad X^s = \{x \in \mathbb{R}^n : \ A^s x \geq b^s\} \qquad f(x) = c^\top x$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in S \\ & \sum_{s \in S} p_s z_s \geq 1 - \tau \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned} \qquad \rightarrow \qquad \begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & z_s = \mathbb{1}\big(A^s x \geq b^s\big), \quad s \in S \\ & \sum_{s \in S} z_s \geq |S| - \ \tau |S| \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned}$$

Linear CCSP with Equiprobable Scenarios

Assumptions

$$p_s = \frac{1}{|S|} \qquad X^s = \{x \in \mathbb{R}^n : \ A^s x \geq b^s\} \qquad f(x) = c^\top x$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in S \\ & \sum_{s \in S} p_s z_s \geq 1 - \tau \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned} \qquad \rightarrow \qquad \begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & z_s = \mathbb{1}\big(A^s x \geq b^s\big), \quad s \in S \\ & \sum_{s \in S} z_s \geq |S| - \lfloor \tau |S| \rfloor \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned}$$

Linear CCSP with Equiprobable Scenarios

Assumptions

$$p_s = \frac{1}{|S|} \qquad X^s = \{x \in \mathbb{R}^n : \ A^s x \geq b^s\} \qquad f(x) = c^\top x$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & f(x) \\ \text{s.t.} \quad & z_s = \mathbb{1}(x \in X^s), \quad s \in S \\ & \sum_{s \in S} p_s z_s \geq 1 - \tau \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned} \qquad \rightarrow \qquad \begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & \boxed{c^\top x} \\ \text{s.t.} \quad & z_s = \mathbb{1}\big(A^s x \geq b^s\big), \quad s \in S \\ & \sum_{s \in S} z_s \geq |S| - \lfloor \tau |S| \rfloor \\ & z_s \in \{0,1\}, \quad s \in S \end{aligned}$$

Challenge 2: Big-M & Relaxation Strength

Reformulation

$$z_s = \mathbb{1}\left(A^s x \geq b^s\right) \qquad \rightarrow \qquad A^s x - b^s \geq \mathrm{M}(1 - z_s)$$

Model

$$\begin{aligned} v^* = \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & A^s x - b^s \geq \mathrm{M}(1 - z_s), \quad s \in S \\ & \sum_{s \in S} z_s \geq |S| - \lfloor \tau |S| \rfloor \\ & z_s \in \{0, 1\}, \quad s \in S \end{aligned}$$

State of the Art: Strengthening Techniques

Cutting planes

Mixing-Set (Luedtke 2014; Song et al. 2014), Quantile (Xie et al. 2018), Simple-Disjunctive (Qiu et al. 2014), Infeasible Irreducible Subsystems (Tanner et al. 2010), Multi-Disjunctive (Cattaruzza et al. 2024)

Big-M tightening

Instance-Specific (Song et al. 2014), Generic (Qiu et al. 2014)

Convex Approximations

Outer (Ahmed et al. 2017), Inner (Jiang et al. 2022; Nemirovski et al. 2007)

Alternative Approaches

Adaptive Partitioning (Roland et al. 2023)

Multi Dimensional Disjunctive Inequalities

Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2$$

Visual Representation



Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2$$

Visual Representation



Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2$$

Visual Representation



Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2$$

Visual Representation

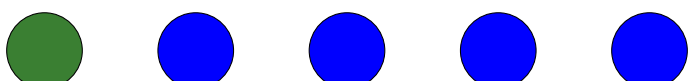


Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2 \qquad |\bar{S}| = 4$$

Visual Representation

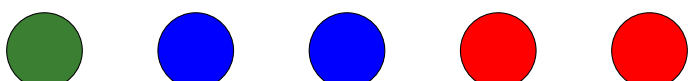


Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2 \qquad |\bar{S}| = 4$$

Visual Representation

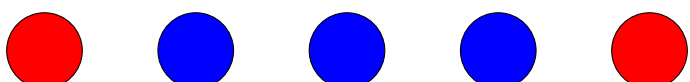


Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2 \qquad |\bar{S}| = 4$$

Visual Representation

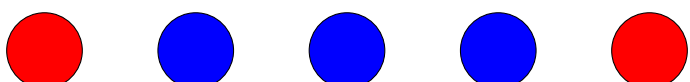


Disjunctive Aspect of CCSPs

Simple Example

$$|S| = 5 \qquad \lfloor \tau |S| \rfloor = 2 \qquad |\bar{S}| = 4$$

Visual Representation



$\longrightarrow$ At least $\max(0, |\bar{S}| - \lfloor \tau |S| \rfloor)$ scenarios are satisfied.

Temporary Assumption

$$A^s x \geq b^s \qquad \rightarrow \qquad a^s x \geq b^s$$

Multi Dimensional Inequalities: Local Validity

Arbitrary Subset

$$\bar{S} \subseteq S \quad \text{where} \quad |\bar{S}| \geq \lfloor \tau |S| \rfloor + 1$$

Multi Dimensional Inequalities: Local Validity

Arbitrary Subset

$$\bar{S} \subseteq S \quad \text{where} \quad |\bar{S}| \geq \lfloor \tau |S| \rfloor + 1$$

Abuse of notation

$$\hat{S}(\bar{x}) = |\bar{S}| - \lfloor \tau |S| \rfloor \text{ scenarios inside } \bar{S}$$

Multi Dimensional Inequalities: Local Validity

Arbitrary Subset

$$\bar{S} \subseteq S \quad \text{where} \quad |\bar{S}| \geq \lfloor \tau |S| \rfloor + 1$$

Abuse of notation

$$\hat{S}(\bar{x}) = |\bar{S}| - \lfloor \tau |S| \rfloor \text{ scenarios inside } \bar{S}$$

Summation

$$\sum_{i \in [n]} \left(\sum_{s \in \hat{S}(\bar{x})} (a_i^s) \right) \bar{x}_i \geq \sum_{s \in \hat{S}(\bar{x})} (b^s)$$

Multi Dimensional Inequalities: Global Validity

Upper Bound

$$\sum_{s \in \hat{S}(\bar{x})} (a_i^s) \leq \text{Greedy Worst Case} = U_i(\bar{S})$$

Multi Dimensional Inequalities: Global Validity

Upper Bound

$$\sum_{s \in \hat{S}(\bar{x})} (a_i^s) \leq \text{Greedy Worst Case} = U_i(\bar{S}) = \max_{w} \quad \sum_{s \in \bar{S}} a_i^s w^s$$

$$\text{s.t.} \quad \sum_{s \in \bar{S}} w^s = |\bar{S}| - \lfloor \tau |S| \rfloor$$

$$w^s \in [0,1], \quad s \in \bar{S}$$

Multi Dimensional Inequalities: Global Validity

Upper Bound

$$\sum_{s \in \hat{S}(\bar{x})} (a_i^s) \leq \text{Greedy Worst Case} = U_i(\bar{S}) = \max_w \quad \sum_{s \in \bar{S}} a_i^s w^s$$

$$\text{s.t.} \quad \sum_{s \in \bar{S}} w^s = |\bar{S}| - \lfloor \tau |S| \rfloor$$

$$w^s \in [0,1], \quad s \in \bar{S}$$

Lower Bound

$$\sum_{s \in \hat{S}(\bar{x})} (b^s) \geq \text{Greedy Worst Case} = L_i(\bar{S}) = \min_w \quad \sum_{s \in \bar{S}} b^s w^s$$

$$\text{s.t.} \quad \sum_{s \in \bar{S}} w^s = |\bar{S}| - \lfloor \tau |S| \rfloor$$

$$w^s \in [0,1], \quad s \in \bar{S}$$

Multi Dimensional Inequalities: Generalization I

Arbitrary subset

$$\bar{S} \subseteq S \quad \text{where} \quad |\bar{S}| \geq \lfloor \tau |S| \rfloor + 1$$

Valid Inequality

$$\sum_{i \in [n]} U_i(\bar{S}) x_i \geq L(\bar{S})$$

Parameters

$$U_i(\bar{S}) = \max_{w} \quad \sum_{s \in \bar{S}} a_i^s w^s$$
$$\text{s.t.} \quad \sum_{s \in \bar{S}} w^s = |\bar{S}| - \lfloor \tau |S| \rfloor$$
$$w^s \in [0,1], \quad s \in \bar{S}$$

$$L(\bar{S}) = \min_{w} \quad \sum_{s \in \bar{S}} b^s w^s$$
$$\text{s.t.} \quad \sum_{s \in \bar{S}} w^s = |\bar{S}| - \lfloor \tau |S| \rfloor$$
$$w^s \in [0,1], \quad s \in \bar{S}$$

Multi Dimensional Inequalities: Generalization II

$$a^s x \geq b^s \qquad \rightarrow \qquad (\lambda^s)^\top A^s x \geq (\lambda^s)^\top b^s$$

Multi Dimensional Inequalities: Summary

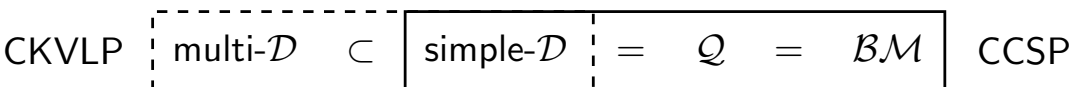
1. Multiplier λ^s

$\rightarrow$ One valid inequality ($\lambda^s A^s x \geq \lambda^s b^s$) for every $s \in \bar{S}$

2. Subset $\bar{S}$

$\rightarrow$ Globally valid upper and lower bound (U_i and L)

Multi Dimensional Inequalities: Closure



Multi Dimensional Inequalities: Exact Separation

- Separation is NP-hard when $|\bar{S}|$ is fixed.

$\rightarrow$ Computing optimal λ^s in P (LP)

$\rightarrow$ Computing good **heuristic** $\bar{S}$ in P (Greedy Algorithm)

Numerical Experiments

Multidimensional Knapsack Instances: Generation

Base Instance



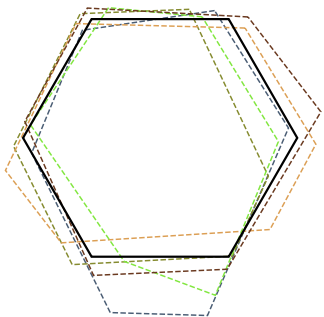
$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Perturbation

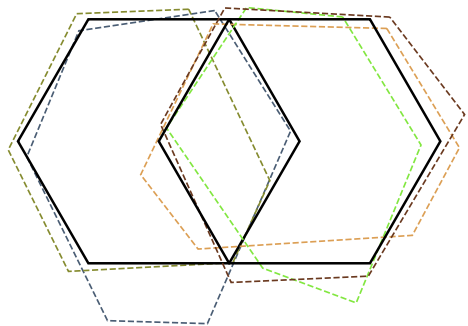
$$a_{ij}^{s} \sim \mathcal{N}(\mu = a_{ij}, \sigma^2 = 0.1 \cdot a_{ij})$$

Multidimensional Knapsack Instances: Visualization

Litterature (`Lit`)



New (`New`)



Cutting Plane Method: Setup

- **Instances**: Perturbed Multidimensional Knapsack
- **Software**: Python, Gurobi, Cython
- **Benchmark**: Quantile and Mixing based on Song et al. 2014

$\rightarrow$ Choose α based on violated constraints

$\rightarrow$ Knapsack specific: tight lower bound on $\rho_s(\alpha)$

Cutting Plane Method: Results

Instances			MD-VIs				Mixing-set			Quantile		
(n, m)	Gen.	$\lvert S\rvert$	s(λ)	T	it.	v (%)	T	it.	v(%)	T	it.	v (%)
mk-20-10	lit	100	0.75	0.23	28.00	**30.97**	0.11	4.80	25.07	0.11	4.20	21.18
		500	0.59	0.50	22.00	**27.27**	0.52	6.20	22.10	0.51	3.80	19.84
		1000	0.74	1.07	28.40	**28.41**	1.09	9.00	24.47	1.04	5.00	21.89
		3000	0.73	3.98	18.80	**27.18**	3.88	17.80	22.99	3.23	4.80	20.82
	new	100	0.31	0.31	46.60	**26.10**	0.12	5.20	16.30	0.12	2.40	0.92
		500	0.33	0.56	49.40	**28.66**	0.60	8.00	18.60	0.58	4.00	1.55
		1000	0.28	1.08	32.80	**27.47**	1.17	8.40	15.92	1.09	4.20	2.65
		3000	0.30	3.78	43.40	**27.25**	3.72	18.20	16.67	3.40	3.80	2.11
mk-40-30	lit	100	0.65	1.12	85.20	**28.84**	1.36	6.40	22.78	1.37	5.60	15.81
		500	0.57	1.59	56.40	**27.92**	6.72	7.40	19.72	6.67	4.60	16.04
		1000	0.64	3.65	108.00	**29.51**	13.55	12.40	22.74	11.83	6.20	18.94
		3000	0.58	12.28	51.00	**27.20**	41.06	17.20	19.05	38.16	5.20	15.85
	new	100	0.22	1.28	130.60	**27.03**	1.25	6.80	17.78	1.37	3.00	0.99
		500	0.25	1.66	91.80	**26.26**	6.89	6.80	14.27	6.74	4.00	1.67
		1000	0.21	3.21	113.40	**26.46**	13.18	10.80	15.73	12.66	4.20	2.14
		3000	0.24	10.67	100.60	**27.29**	40.25	19.80	14.93	36.87	4.00	1.77

Cutting Plane Method + MIP: Setup

- **Instances**: Perturbed Multidimensional Knapsack
- **x Variables**: Binary
- **Software**: Python, Gurobi, Cython
- **Time**: 2 hours

Cutting Plane Method + MIP: Results

(n, m)	Gen.	$\lvert S\rvert$	MIP	MD-VIs	Mixing-set	Quantile
mk-20-10	lit	100	9.28s	11.26s	6.34s	**5.49s**
		500	110.42s	**33.71s**	60.40s	39.13s
		1000	642.60s	**110.09s**	260.68s	164.71s
		3000	12.03%(1)	1631.90s	4.60%(4)	**1500.19s**
	new	100	14.43s	22.39s	**8.01s**	13.11s
		500	91.75s	**86.46s**	72.04s	91.51s
		1000	388.66s	**222.53s**	356.02s	442.39s
		3000	41.46%(0)	**12.50%(2)**	37.82%(0)	52.73%(0)
mk-40-30	lit	100	10.17%(0)	**6.54%(1)**	11.73%(0)	12.05%(0)
		500	29.22%(0)	**3111.45s**	14.44%(0)	15.76%(0)
		1000	52.17%(0)	**17.08%(0)**	29.35%(0)	34.95%(0)
		3000	41.63%(0)	**12.77%(0)**	24.66%(0)	29.13%(0)
	new	100	19.70%(0)	20.58%(0)	21.98%(0)	**19.05%(0)**
		500	39.24%(0)	**15.61%(0)**	27.21%(0)	34.72%(0)
		1000	48.89%(0)	**25.55%(0)**	39.56%(0)	52.94%(0)
		3000	49.05%(0)	**25.79%(0)**	42.26%(0)	50.00%(0)

Conclusion

Problem: Linear Chance Constrained Stochastic Programs with Finite Support

Challenges: Non-convexity, Weak LP relaxations

State of the Art: Quantile and Mixing Set Inequalities

Concept: Disjunctive Aspect of CCSPs

Theoretical Dominance:

$$\text{multi-}\mathcal{D} \subset \mathcal{Q} = \mathcal{BM}$$

Computational Dominance: Better performance + More robust