Adaptive Nonlinear Optimization of District Heating Networks Based on Model and Discretization Hierarchies

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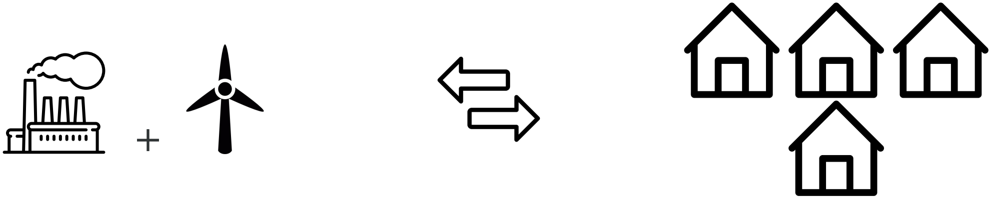
Trier University

The Energy Transition

“Heating and cooling in buildings and industry accounts for **half** of the EU’s energy consumption.”

- Use cleaner energy
- Distribute energy efficiently
- Store energy

$\Rightarrow$ District Heating Networks



District Heating Networks and Optimization

Optimization

- Control (Benonysson et al. 1995; Krug et al. 2020; Sandou et al. 2005; Verrilli et al. 2017)
- Design (Ameri and Besharati 2016; Blommaert et al. 2018; Bracco et al. 2013; Guelpa et al. 2018; Mertz et al. 2017; Söderman 2007)
- Expansion (Bordin et al. 2016)

Challenges

- Nonlinear behaviour of fluids in pipes
- Mixing of water at pipe intersections
- Water flow directions
- Modeling accuracy
- Discretization accuracy

Assumptions

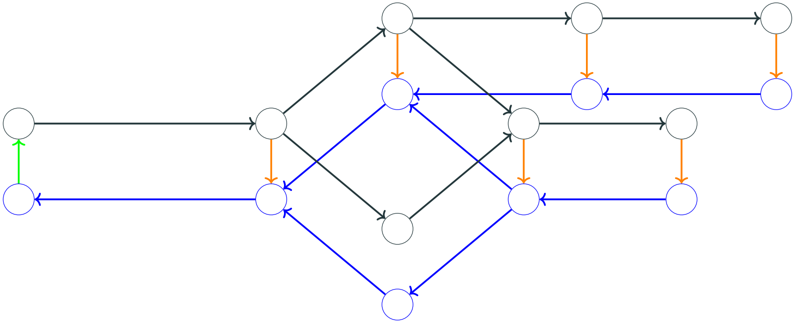
- One dimensional stationary model

Overview

Operational Optimization Problem

$$\begin{aligned} \min \quad & \text{Operation Cost} \\ \text{s.t.} \quad & \text{System physics} \\ & \text{Consumer constraints} \\ & \text{Depot constraints} \end{aligned}$$

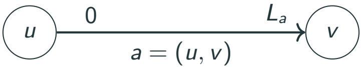
Network Modeling and Notation



We introduce the graph $G = (V, A)$ such that

- A_{ff}, A_{bf}, A_{c}, a_{d}
- V_{ff}, V_{bf}

Pipe Physics



One-dimensional stationary momentum equation for incompressible fluids in cylindrical pipes (Hauschild et al. 2020; Krug et al. 2020)

$$\frac{p_a(L_a) - p_a(0)}{L_a} = -g\rho h_a' - \lambda_a \frac{|v_a| v_a \rho}{2D_a}, \quad a \in A_{\text{pi}}$$

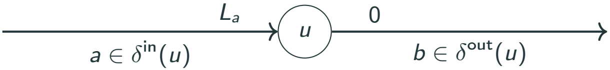
One-dimensional stationary internal energy conservation (Hauschild et al. 2020)

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} - \frac{\lambda_a}{2D_a}\rho |v_a| v_a^2 + \frac{4U}{D_a}(T_a - T_{\text{soil}}), \quad a \in A_{\text{pi}}$$

State equations (ibid.)

$$\left.\begin{aligned} \rho &= 997\,\mathrm{kg\,m^{-3}}, \\ T_a &= \theta_2 (e_a)^2 + \theta_1 e_a + \theta_0, \end{aligned}\right. \quad a \in A_{\text{pi}}$$

Node Physics



Mass flow continuity

$$\sum_{a \in \delta^{\text{in}}(u)} q_a = \sum_{a \in \delta^{\text{out}}(u)} q_a$$

Pressure continuity

$$p_u = p_a(0), \quad a \in \delta^{\text{out}}(u)$$
$$p_u = p_a(L_a), \quad a \in \delta^{\text{in}}(u)$$

Internal energy mixing (Krug et al. 2020)

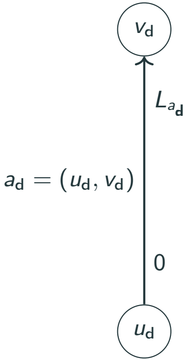
$$\sum_{a \in \delta^{\text{in}}(u)} \frac{e_a(L_a) q_a}{\rho_a} = \sum_{a \in \delta^{\text{out}}(u)} \frac{e_a(0) q_a}{\rho_a}$$

$$0 = \beta_a(e_a(0) - e_u), \quad a \in \delta^{\text{out}}(u)$$
$$0 = \gamma_a(e_a(L_a) - e_u), \quad a \in \delta^{\text{in}}(u)$$

$$\left.\begin{aligned} q_a &= \beta_a - \gamma_a, \\ \beta_a &\perp \gamma_a, \end{aligned}\right. \quad a \in \delta(u)$$

For $a_\mathbf{d} = (u_\mathbf{d}, v_\mathbf{d})$

$$
\begin{aligned}
p_u &= p_s \\
P_p &= \frac{q_{a_\mathbf{d}}}{\rho_{a_\mathbf{d}}} \left(p_{a_\mathbf{d}:v} - p_{a_\mathbf{d}:u}\right) \\
P_w + P_g &= \frac{q_{a_\mathbf{d}}}{\rho_{a_\mathbf{d}}} \left(e_{a_\mathbf{d}:v} - e_{a_\mathbf{d}:u}\right)
\end{aligned}
$$



Consumers

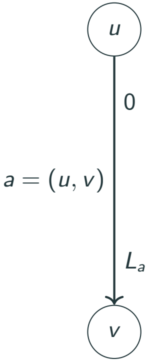
For $a = (u, v) \in A_{\mathsf{c}}$

$$P_a = \frac{q_a}{\rho_a}\left(e_{a:v} - e_{a:u}\right)$$

$$e_{a:u} \geq e_a^{\mathsf{ff}}$$

$$e_{a:v} = e^{\mathsf{bf}}$$

$$p_v \leq p_u$$



Objective

$$\min \quad C_{\mathsf{p}} P_p + C_{\mathsf{w}} P_w + C_{\mathsf{g}} P_g$$

Model Summary

min objective

s.t. stationary momentum equation

stationary internal energy conservation

mass conservation

pressure continuity

internal energy mixing equations

depot constraints

consumer constraints

system bounds

power bounds

Analytical Solution

If $T_{\text{soil}} \in [228, \infty)$, then the differential equation,

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} - \frac{\lambda_a}{2D_a}\rho|v_a|v_a^2 + \frac{4U}{D_a}(T_a - T_{\text{soil}}),$$

with initial condition

$$e_a(0) = e_a^{\mathbf{0}} > 0,$$

and state equation

$$T_a = \theta_2(e_a)^2 + \theta_1 e_a + \theta_0,$$

has the solution

$$e_a(x) = \frac{\sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \frac{1 + \exp\left(\frac{x\sqrt{\beta^2-4\alpha\gamma}}{\zeta}\right)\left(\frac{2\alpha e_a^{\mathbf{0}}+\beta-\sqrt{\beta^2-4\alpha\gamma}}{2\alpha e_a^{\mathbf{0}}+\beta+\sqrt{\beta^2-4\alpha\gamma}}\right)}{1 - \exp\left(\frac{x\sqrt{\beta^2-4\alpha\gamma}}{\zeta}\right)\left(\frac{2\alpha e_a^{\mathbf{0}}+\beta-\sqrt{\beta^2-4\alpha\gamma}}{2\alpha e_a^{\mathbf{0}}+\beta+\sqrt{\beta^2-4\alpha\gamma}}\right)} - \frac{\beta}{2\alpha},$$

with

$$\alpha := -\frac{4U\theta_2}{D_a(e_0)^2}, \quad \beta := -\frac{4U\theta_1}{D_a e_0}, \quad \zeta := v_a,$$

$$\gamma := \frac{\lambda_a}{2D_a}\rho|v_a|v_a^2 - \frac{4U}{D_a}(\theta_0 - T_{\text{soil}}).$$

Discretized Solution

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} - \frac{\lambda_a}{2D_a}\rho|v_a|v_a^2 + \frac{4U}{D_a}(T_a - T_{\text{soil}})$$

with state equation

$$T_a = \theta_2(e_a)^2 + \theta_1 e_a + \theta_0$$

$$\Downarrow$$

$$0 = v_a \left(\frac{e_a(x_k) - e_a(x_{k-1})}{\Delta x} \right) - \frac{\lambda_a \rho}{2D_a}|v_a|v_a^2 + \frac{4U}{D_a}\left(T_a(e_a(x_k), e_a(x_{k-1})) - T_{\text{soil}}\right)$$

with state equation

$$T_a(e_a(x_k), e_a(x_{k-1})) := \frac{\theta_2}{4}\left(e_a(x_k) + e_a(x_{k-1})\right)^2 + \frac{\theta_1}{2}\left(e_a(x_k) + e_a(x_{k-1})\right) + \theta_0$$

Model Summary

min	objective	
s.t.	stationary momentum equation	
	discretized stationary internal energy conservation	
	mass conservation	
	pressure continuity	
	internal energy mixing equations	(NLP)
	depot constraints	
	consumer constraints	
	system bounds	
	power bounds	

```
 Iter Phase   Ninf   Infeasibility   RGmax      NSB   Step  InItr MX OK
  380   2      368  3.1815212151E+01 9.8E-01     73 8.6E-01    14 F  F
  381   2      366  3.1815207829E+01 2.6E-01     73 1.0E+00    16 T  T
  382   1      365  3.1815207347E+01 5.8E-01     74 1.0E+00     2 T  T
  383   1      365  3.1815207329E+01 2.9E-07     24 1.0E+00     1 T  T
  384   2      365  3.1815207329E+01 0.0E+00    334 0.0E+00       T  T
  385   2      365  3.1815207329E+01 0.0E+00    334 0.0E+00       T  T
  386   2      365  3.1815207329E+01 0.0E+00    334 0.0E+00       T  T
  387   2      365  3.1815207329E+01 0.0E+00    333 0.0E+00       T  T
  388   2      365  3.1815207329E+01 0.0E+00    333 0.0E+00       T  T
  389   2      365  3.1815207329E+01 0.0E+00    333 0.0E+00       T  T

 Iter Phase   Ninf   Infeasibility   RGmax      NSB   Step  InItr MX OK
  390   1      365  3.1815207329E+01 0.0E+00     88

** Infeasible solution. Reduced gradient less than tolerance.
```

Issues

- Solver does not converge
- How detailed do we have to be?

Causes

- Incorrect discretization
- Model too detailed
- No error measure

Idea's

- Discretization hierarchies
- Model hierarchies
- Error measures
- Termination condition

Model Hierarchy

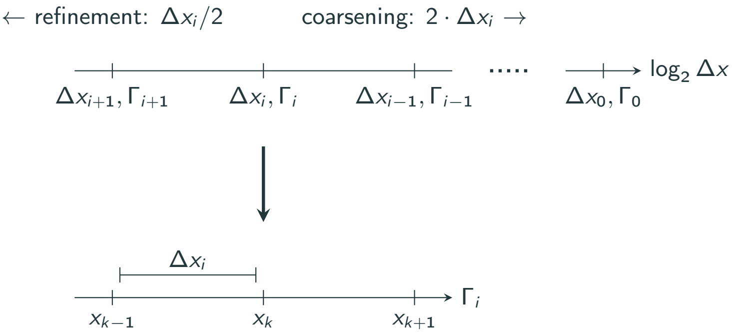
$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} - \frac{\lambda_a}{2D_a} \rho_a |v_a| v_a^2 + \frac{4U}{D_a}(T_a - T_{\text{soil}}) \tag{M$_1$}$$

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} - \frac{\lambda_a}{2D_a} \rho_a v_a^2 |v_a| + \frac{4U}{D_a}(T_a - T_{\text{soil}}) \tag{M$_2$}$$

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} + \frac{4U}{D_a}(T_a - T_{\text{soil}}) \tag{M$_2$}$$

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} + \frac{4U}{D_a}(T_a - T_{\text{soil}}), \tag{M$_3$}$$

$$0 = v_a \frac{\mathrm{d}e_a}{\mathrm{d}x} \tag{M$_3$}$$


$\leftarrow$ refinement: $\Delta x_i/2$
coarsening: $2 \cdot \Delta x_i \rightarrow$
$\log_2 \Delta x$
$\Delta x_{i+1}, \Gamma_{i+1}$
$\Delta x_i, \Gamma_i$
$\Delta x_{i-1}, \Gamma_{i-1}$
$\Delta x_0, \Gamma_0$
Δx_i
Γ_i
x_{k-1}
x_k
x_{k+1}

Error Measures

Exact solution

$$e_a^{\ell_a}(\Gamma_0) := [e_a^{\ell_a}(x_1), \dots, e_a^{\ell_a}(x_n)]^\top$$

Approximate solution

$$e_a^{\ell_a}(\Gamma_0; \Delta x_a) := [e_a^{\ell_a}(x_1; \Delta x_a), \dots, e_a^{\ell_a}(x_n; \Delta x_a)]^\top$$

Exact total error

$$\nu_a(y) := \|e_a^1(\Gamma_0) - e_a^{\ell_a}(\Gamma_0; \Delta x_a)\|_\infty$$

Exact model error

$$\nu_a^{\mathsf{m}}(y) := \|e_a^1(\Gamma_0) - e_a^{\ell_a}(\Gamma_0)\|_\infty$$

Exact discretization error

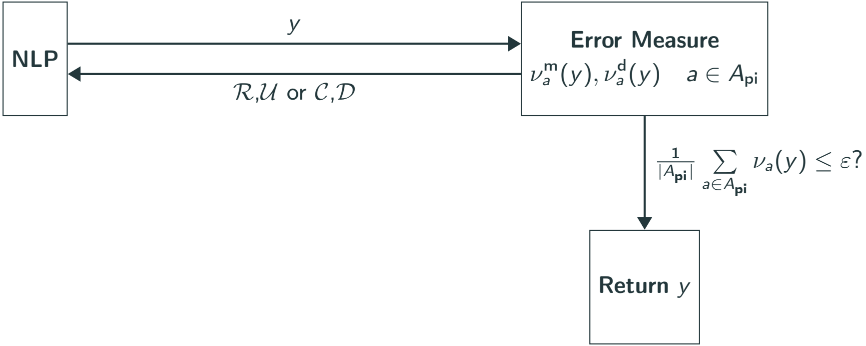
$$\nu_a^{\mathsf{d}}(y) := \|e_a^{\ell_a}(\Gamma_0) - e_a^{\ell_a}(\Gamma_0; \Delta x_a)\|_\infty$$

Upper bound

$$\begin{aligned}
\nu_a(y) &= \|e_a^1(\Gamma_0) - e_a^{\ell_a}(\Gamma_0; \Delta x_a) + e_a^{\ell_a}(\Gamma_0) - e_a^{\ell_a}(\Gamma_0)\|_\infty \\
&\leq \|e_a^1(\Gamma_0) - e_a^{\ell_a}(\Gamma_0)\|_\infty + \|e_a^{\ell_a}(\Gamma_0) - e_a^{\ell_a}(\Gamma_0; \Delta x_a)\|_\infty \\
&= \nu_a^{\mathsf{m}}(y) + \nu_a^{\mathsf{d}}(y)
\end{aligned}$$

Let $\varepsilon > 0$ be a given tolerance. We say that a solution y of problem (NLP) with discretized models M_{ℓ_a}, $\ell_a \in \{1, 2, 3\}$, and stepsizes Δx_a for the pipes $a \in A_{\mathbf{pi}}$ is ε-feasible with respect to the reference problem if The solution y of the (NLP) is called ε-feasible if

$$\frac{1}{|A_{\mathbf{pi}}|} \sum_{a \in A_{\mathbf{pi}}} \nu_a(y) \leq \varepsilon.$$


NLP
y
Error Measure
$\nu_a^{\mathrm{m}}(y), \nu_a^{\mathrm{d}}(y) \quad a \in A_{\mathbf{pi}}$
$\mathcal{R}$,$\mathcal{U}$ or $\mathcal{C}$,$\mathcal{D}$
$\frac{1}{|A_{\mathbf{pi}}|} \sum_{a \in A_{\mathbf{pi}}} \nu_a(y) \leq \varepsilon$?
Return y

Algorithm Pseudocode

Algorithm 1: Adaptive Model and Discretization Control

Input: Network (V, A), intial- and boundary conditions, error tolerance $\varepsilon > 0$, initial parameters $\Theta_{\mathcal{R}}, \Theta_{\mathcal{U}}, \Theta_{\mathcal{C}}, \Theta_{\mathcal{D}} \in (0,1)$, $\tau^0 \leq 1$, $\mu^0 \in \mathbb{N}_+$

Output: ε-feasible solution y of (NLP)

for $a \in A_{pi}$ **do**
    initialize model level $\ell_a^{0,0}$ and step size $\Delta x_a^{0,0}$
end
for $k = 0, 1, 2, \ldots$ **do**
    for $j = 0, \ldots, \mu^k$ **do**
        if $k > 0$ *and* $j > 0$ **then**
            compute sets $\mathcal{U}^{k,j}, \mathcal{R}^{k,j} \subseteq A_{\text{pi}}$ and apply their respective changes
        end
        $y^{k,j} \leftarrow$ solve (NLP)
        if $y^{k,j}$ *is* ε*-feasible* **then** return $y^{k,j}$.
    end
    compute sets $\mathcal{D}^{k,j}, \mathcal{C}^{k,j} \subseteq A_{\text{pi}}$ and apply their respective changes
    $y^{k,j} \leftarrow$ solve (NLP)
    if $y^{k,j}$ *is* ε*-feasible* **then** return $y^{k,j}$
end

We determine $\mathcal{R}$ and $\mathcal{U}$ by finding the **minimum** subset of pipes $a \in A_{\mathsf{pi}}$ such that

$$\Theta_{\mathcal{R}} \sum_{a \in A_{\mathsf{pi}}} \nu_a^{\mathsf{d}}(y) \leq \sum_{a \in \mathcal{R}} \nu_a^{\mathsf{d}}(y)$$

and

$$\Theta_{\mathcal{U}} \sum_{a \in A_{\mathsf{pi}}^{>\varepsilon}} \left(\nu_a^{\mathsf{m}}(y; \ell_a) - \nu_a^{\mathsf{m}}(y; \ell_a^{\mathsf{new}})\right) \leq \sum_{a \in \mathcal{U}} \left(\nu_a^{\mathsf{m}}(y; \ell_a) - \nu_a^{\mathsf{m}}(y; \ell_a^{\mathsf{new}})\right)$$

We determine $\mathcal{C}$ and $\mathcal{D}$ find the **maximum** subset of all pipes $a \in A_{\mathsf{pi}}$ such that

$$\Theta_{\mathcal{C}} \sum_{a \in A_{\mathsf{pi}}} \nu_a^{\mathsf{d}}(y) \geq \sum_{a \in \mathcal{C}} \nu_a^{\mathsf{d}}(y)$$

and

$$\Theta_{\mathcal{D}} \sum_{a \in A_{\mathsf{pi}}^{<\varepsilon}} \left(\nu_a^{\mathsf{m}}(y; \ell_a^{\mathsf{new}}) - \nu_a^{\mathsf{m}}(y; \ell_a)\right) \geq \sum_{a \in \mathcal{D}} \left(\nu_a^{\mathsf{m}}(y; \ell_a^{\mathsf{new}}) - \nu_a^{\mathsf{m}}(y; \ell_a)\right)$$

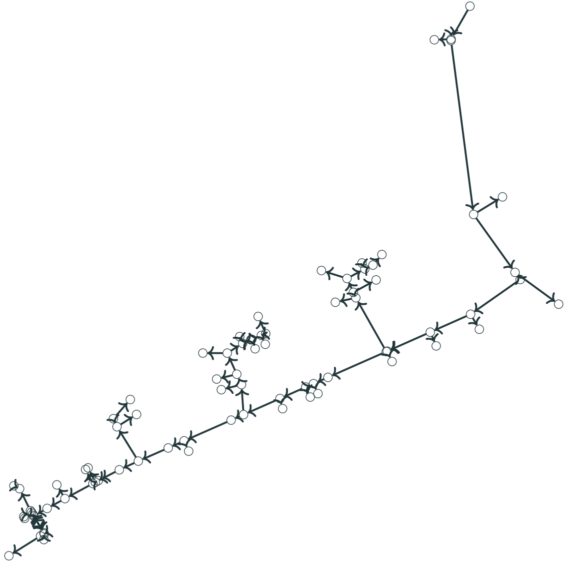
Theorem (Finite Termination)

Suppose that $\nu_a^d \ll \nu_a^m$ for every $a \in A_{pi}$ and that every NLP is solved to local optimality. Then, Algorithm 1 terminates after a finite number of refinements, coarsenings and model switches with an ε-feasible solution with respect to the reference problem if there exist constants $C_1, C_2 > 0$ such that

$$\frac{1}{4}\Theta_{\mathcal{R}}\mu \geq \Theta_{\mathcal{C}} + C_1, \quad \Theta_{\mathcal{U}}\mu \geq \tau\Theta_{\mathcal{D}}|A_{pi}| + C_2,$$

hold.

Test Case: Network



Characteristics

- 162 pipes
- 32 consumers
- 1 depot

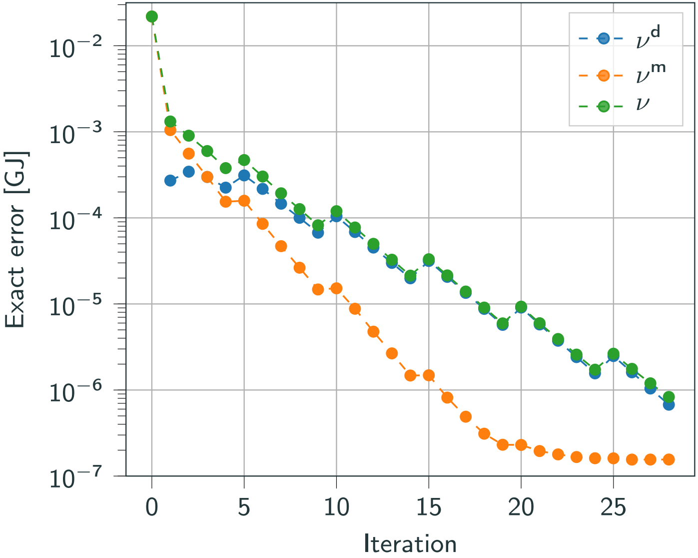
Test Case: Parameters

Parameter	Value
ε	10^{-6} GJ
$\Theta_{\mathcal{R}}$	0.7
$\Theta_{\mathcal{U}}$	0.4
$\Theta_{\mathcal{C}}$	0.6
$\Theta_{\mathcal{D}}$	0.01
τ	1.1
μ	4

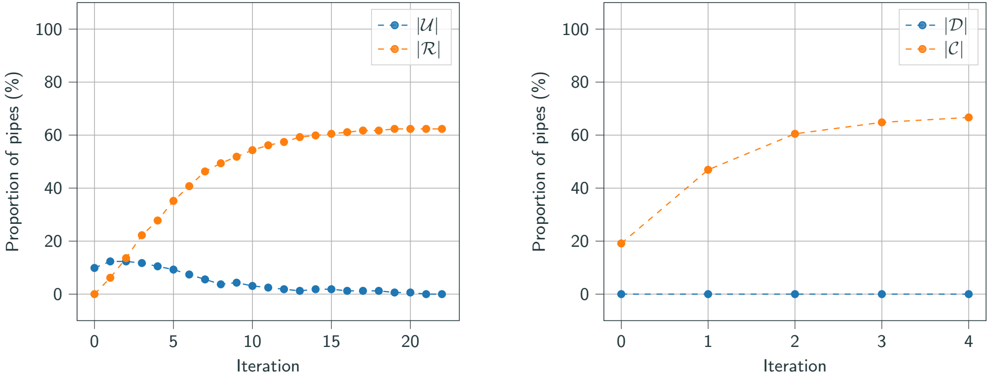
Cost	C_w	C_g	C_p
Value (€/kWh)	0	0.0415	0.165

Intel(R) Core(TM) i7-8550U, 16 GB RAM.
CONOPT4 using the Pyomo interface.

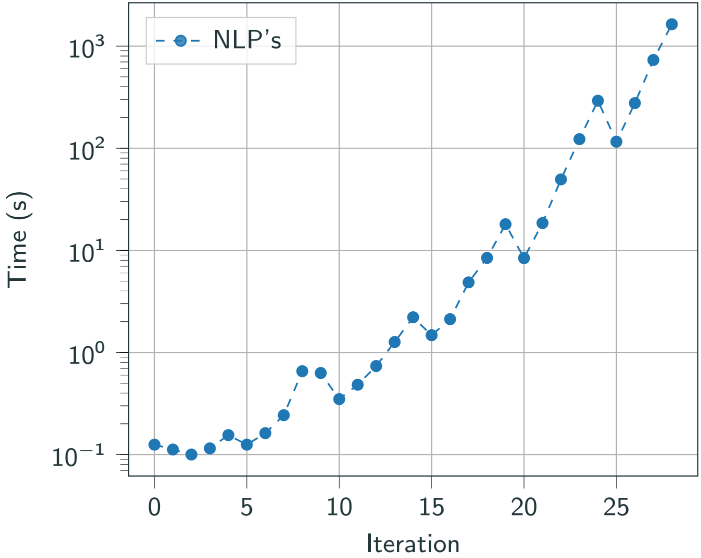
Error Measures



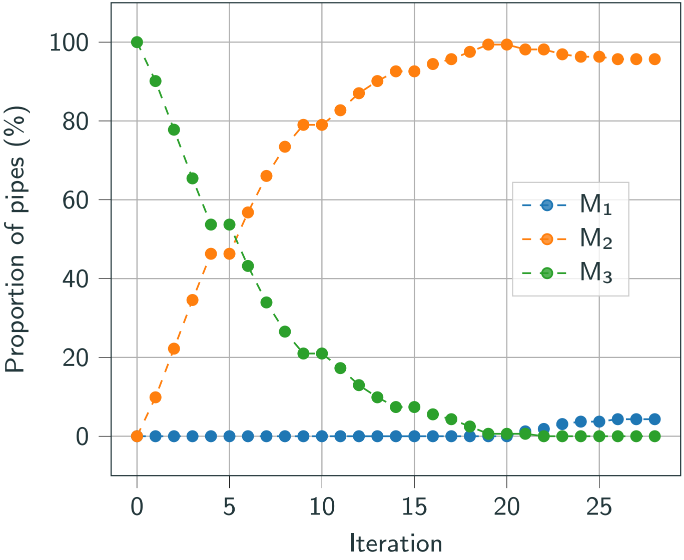
Subset Sizes



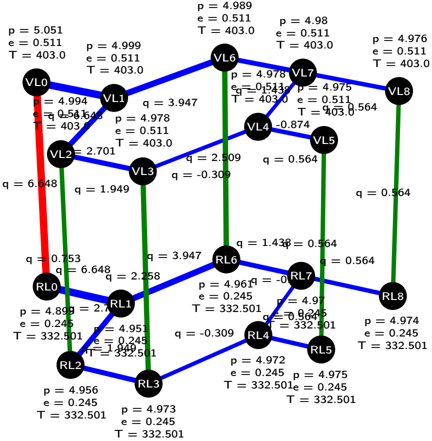
Model Level Sets Sizes

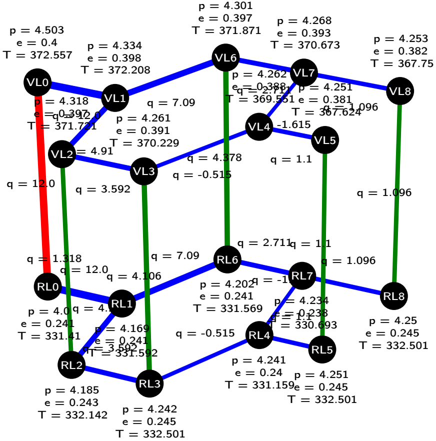


Computation Time



Smaller Test Case





Conclusion

Contributions

- Model hierarchy for the internal energy conservation equation.
- Coupled with error measures
- Adaptive optimization algorithm

Results

- Prove finite termination under reasonable assumptions
- To ε-feasible solutions
- Solve problem instances not solvable before

Next steps

- Time dependent case
- Larger networks
- Other problems

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