Mixed-Integer Nonlinear Optimization for District Heating Network Expansion

Marius Roland, Martin Schmidt

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Trier University

The Energy Transition

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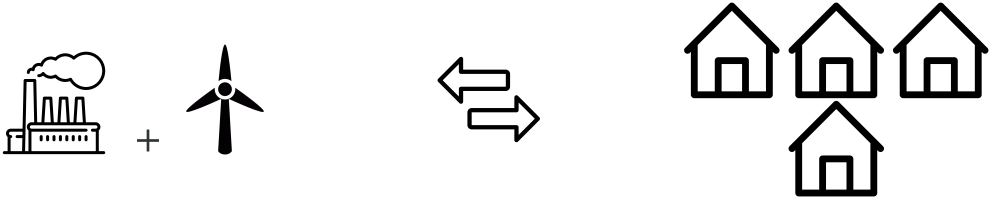

Optimization

- Control (Benonysson et al. 1995; Krug et al. 2020; Sandou et al. 2005; Verrilli et al. 2017)
- Design (Ameri and Besharati 2016; Blommaert et al. 2018; Bracco et al. 2013; Guelpa et al. 2018; Mertz et al. 2017; Söderman 2007)
- Expansion (Bordin et al. 2016)

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Challenges

- Nonlinear behaviour of fluids in pipes
- Mixing of water at pipe intersections
- Water flow directions
- Modeling of **candidate** consumers

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Assumptions

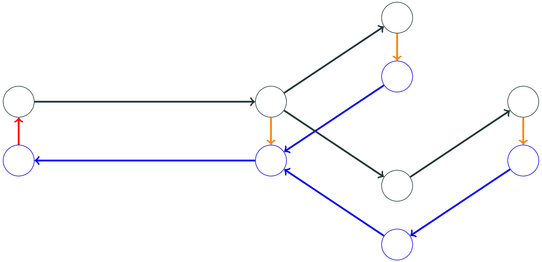
- One dimensional stationary model
- Only tree shaped networks

Overview

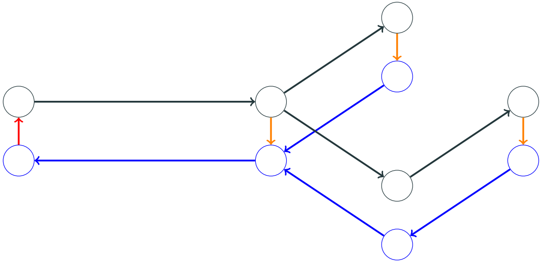
Expansion Problem

$$
\begin{aligned}
\max \quad & \text{Profit} = \text{Consumer Payment - Expansion, Operation and Maintenance Cost} \\
\text{s.t.} \quad & \text{System physics} \\
& \text{Consumer constraints} \\
& \text{Depot constraints}
\end{aligned}
$$

Network Modeling and Notation



Network Modeling and Notation



We introduce the graph $G = (V, A)$ such that

- A_{ff}, A_{bf}, A_{c}, a_{d}
- V_{ff}, V_{bf}

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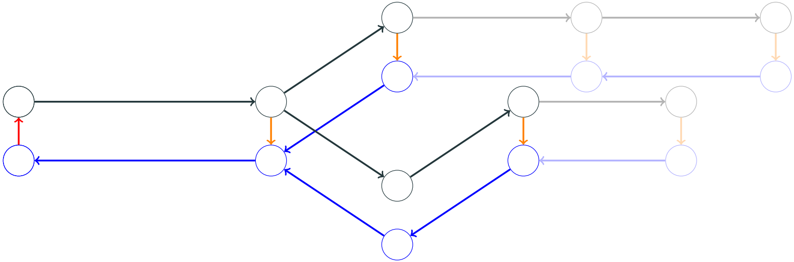

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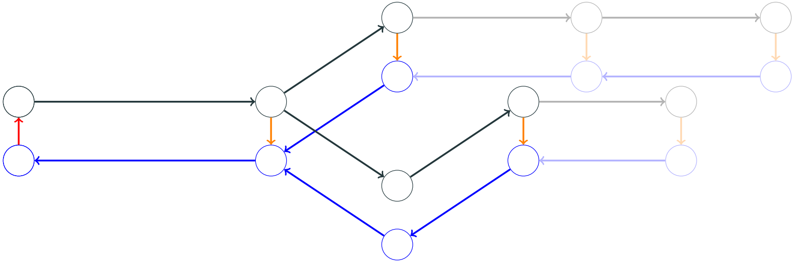
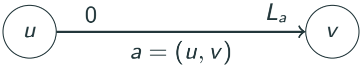


We introduce the graph $G = (V, A)$ such that

- A_{ff}, A_{bf}, A_{c}, a_{d}
- V_{ff}, V_{bf}
- A_{ff}^{e}, A_{ff}^{c}, A_{bf}^{e}, A_{bf}^{c}, A_{c}^{e}, A_{c}^{c}
- V_{ff}^{e}, V_{ff}^{c}, V_{bf}^{e}, V_{bf}^{c}

Pipe Physics



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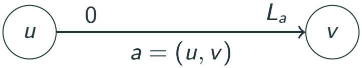

One-dimensional stationary momentum equation for compressible fluids in cylindrical pipes (Hauschild et al. 2020; Krug et al. 2020)

$$\frac{p_a(L_a) - p_a(0)}{L_a} = -g\rho h'_a - \lambda_a \frac{|v_a| v_a \rho}{2D_a}, \quad \forall a \in A_{\mathrm{ff}} \cup A_{\mathrm{bf}}$$

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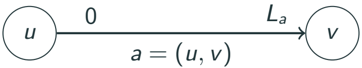

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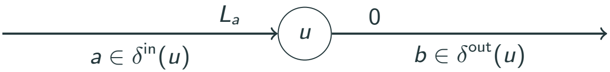
One-dimensional stationary thermal energy equation

$$T_a(L_a; v_a) = \begin{cases} T_{\text{soil}}, & v_a = 0 \\ (T_a(0) - T_{\text{soil}})\, e^{-\frac{4U_a L_a}{c_p \rho D_a v_a}} + T_{\text{soil}}, & v_a > 0 \end{cases}, \quad \forall a \in A_{\text{ff}} \cup A_{\text{bf}}$$

Node Physics



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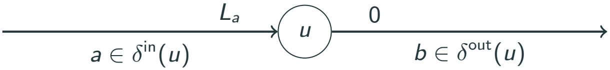
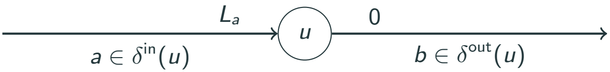


Mass flow continuity

$$\sum_{a \in \delta^{\text{in}}(u)} q_a = \sum_{a \in \delta^{\text{out}}(u)} q_a, \quad u \in V$$

Node Physics



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Pressure continuity

$$p_u = p_a(0), \quad u \in V, \; a \in \delta^{\text{out}}(u)$$
$$p_u = p_a(L_a), \quad u \in V, \; a \in \delta^{\text{in}}(u)$$

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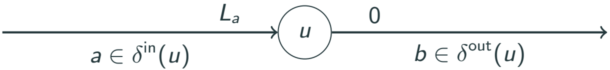

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Temperature mixing (Krug et al. 2020)

$$T_u = \frac{\sum_{a \in \delta^{\text{in}}(u)} c_p q_a T_a(L_a)}{\sum_{a \in \delta^{\text{in}}(u)} c_p q_a}, \quad u \in V$$

$$T_u = T_a(0), \quad u \in V, \ a \in \delta^{\text{out}}(u)$$

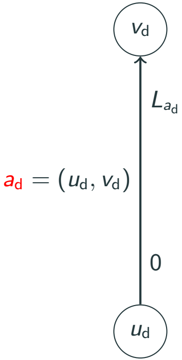
Depot

For $a_\mathrm{d} = (u_\mathrm{d}, v_\mathrm{d})$

$$p_{u_\mathrm{d}} = p_s$$

$$P_p = \frac{q_{a_\mathrm{d}}}{\rho} (p_{v_\mathrm{d}} - p_{u_\mathrm{d}})$$

$$P_w + P_g = q_{a_\mathrm{d}} c_p (T_a(L_{a_\mathrm{d}}) - T_a(0))$$



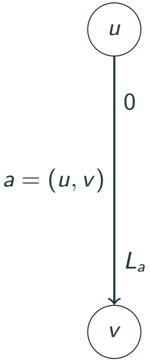
$$P_a = q_a c_p \left(T_a(0) - T_a(L_a) \right), \qquad a \in A_c^e$$

$$x_a P_a = q_a c_p \left(T_a(0) - T_a(L_a) \right), \qquad a \in A_c^c$$

$$T_a(L_a) = T^{bf}, \qquad a \in A_c^e \cup A_c^c$$

$$T_a(0) \geq T_a^{ff}, \qquad a \in A_c^e \cup A_c^c$$

$$p_v \leq p_u, \qquad a \in A_c^e \cup A_c^c$$



Thermal Energy Equation Approximation I

$$f(v_a, T_a(0), T_a(L_a)) := \begin{cases} \left(T_a(0) - T_{\text{soil}}\right) e^{-\frac{4U_a L_a}{c_p \rho D_a v_a}} + T_{\text{soil}} - T_a(L_a), & v_a > 0 \\ T_{\text{soil}} - T_a(L_a), & v_a = 0 \end{cases}$$

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$$\Downarrow$$

$$f_{\text{approx}}(v_a, T_a(0), T_a(L_a)) := \sum_{(k,l,m)\in\Theta_d} \alpha_{klm}\, v_a^k\, T_a(0)^l\, T_a(L_a)^m + T_a(L_a) - T_{\text{soil}},$$

with

$$\Theta_d := \left\{(k, l, m) \in \mathbb{N}^3 \colon\ k \neq 0 \text{ and } k + l + m \leq d\right\}.$$

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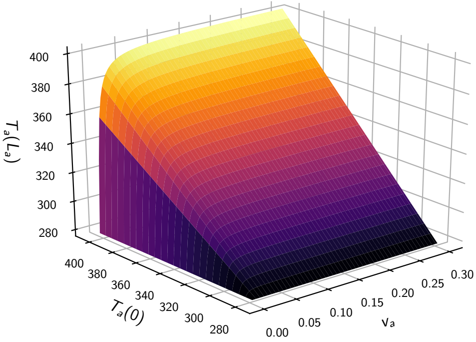
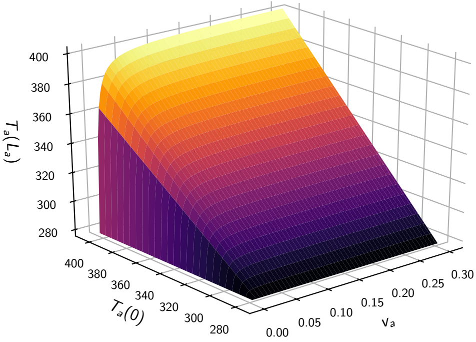
$$\Downarrow$$

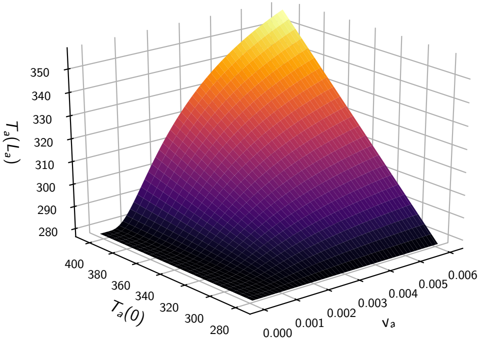
$$\min_{\alpha} \quad \sum_{i\in I} f_{\text{approx}}(v_a^i, T_a(0)^i, T_a(L_a)^i)^2$$

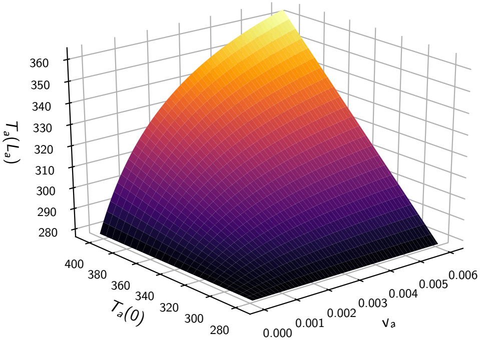
Thermal Energy Equation Approximation II



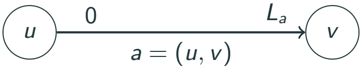


Thermal Energy Equation Approximation III





Candidate Pipe Modeling: Pressure Constraints



One-dimensional stationary momentum equation for compressible fluids in cylindrical pipes

$$p_v - p_u + L_a g \rho h'_a + \lambda_a \frac{|v_a| v_a \rho L_a}{2D_a} = 0, \quad \forall a \in A_{\text{ff}} \cup A_{\text{bf}}$$

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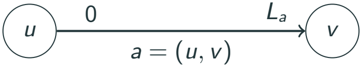

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$$p_v - p_u + L_a g \rho h'_a + \lambda_a \frac{|v_a| v_a \rho L_a}{2 D_a} = 0, \quad \forall a = (u, v) \in A^{e}_{ff} \cup A^{e}_{bf}$$

And,

$$p_v - p_u + L_a g \rho h'_a + \lambda_a \frac{|v_a| v_a \rho L_a}{2 D_a} \leq (1 - x_a) M^1_a, \quad \forall a = (u, v) \in A^{c}_{ff} \cup A^{c}_{bf}$$

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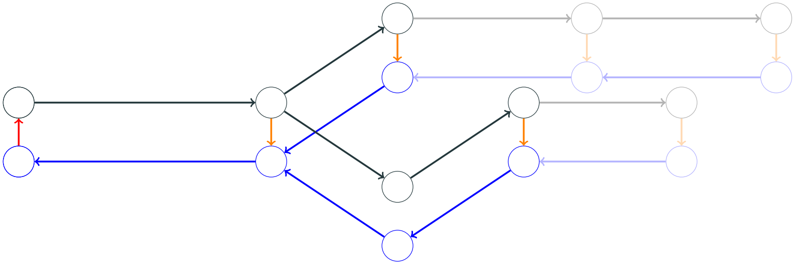
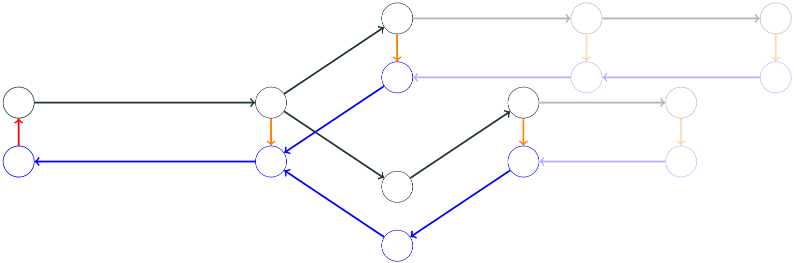
Candidate Pipe Modeling: Valid Inequalities





$$x_{a_1} \leq x_{a_2}, \quad a_1, a_2 \in A_{\mathrm{ff}}^{\mathrm{c}} \cup A_{\mathrm{bf}}^{\mathrm{c}} \cup A_{\mathrm{c}}^{\mathrm{c}}, \quad a_2 \in P(a_1)$$

Objective

$$\max \sum_{a \in A^{\mathrm{c}}_{\mathrm{c}}} P_a w \pi x_a - \sum_{a \in A^{\mathrm{c}}_{\mathrm{ff}} \cup A^{\mathrm{c}}_{\mathrm{bf}} \cup A^{\mathrm{c}}_{\mathrm{c}}} C^{\mathrm{inv}}_a x_a - w\left(C_{\mathrm{p}} P_p + C_{\mathrm{w}} P_w + C_{\mathrm{g}} P_g\right)$$

Model Summary

max objective

s.t. stationary incompressible Euler equation

stationary thermal energy equation approximation

mass conservation

pressure continuity,

temperature mixing equations

depot constraints

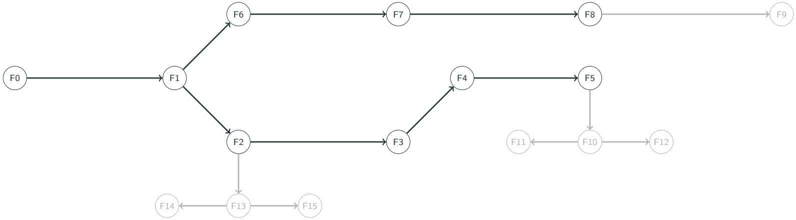
consumer constraints

system bounds

power bounds

valid binary inequalities

Test Case



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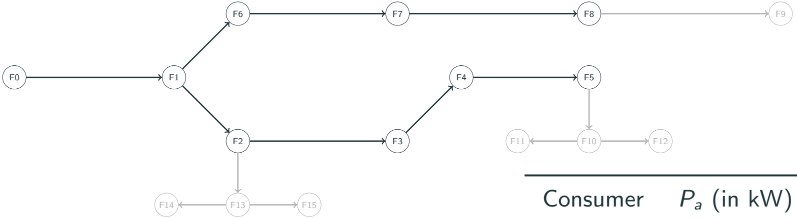
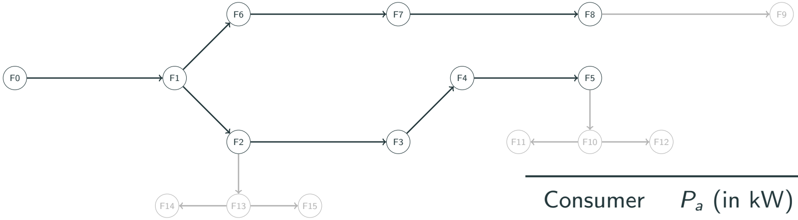


Cost	C_w	C_g	C_p
Value (€/kWh)	0	0.0415	0.165

Edge	$a \in A_c^c$	$a \in A_{ff}^c \cup A_{bf}^c$
C_a^{inv} (€)	100 000	[90 000, 330 000]

Consumer	P_a (in kW)
(F2,B2)	200.00
(F3,B3)	600.00
(F5,B5)	150.00
(F6,B6)	666.66
(F8,B8)	200.00
(F9,B9)	183.33
(F11,B11)	183.33
(F12,B12)	183.33
(F14,B14)	183.33
(F15,B15)	183.33

Test Case



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C_a^{inv} (€)	100 000	[90 000, 330 000]

Intel(R) Core(TM) i7-8550U, 16 GB RAM.
ANTIGONE using the Pyomo interface.

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(F2,B2)	200.00
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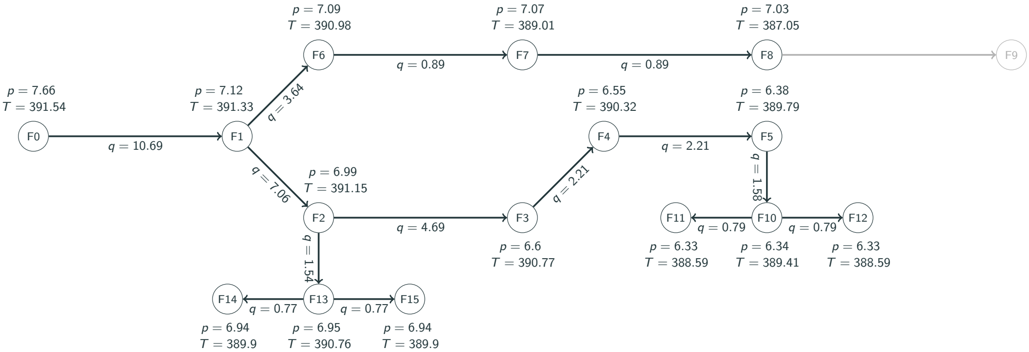
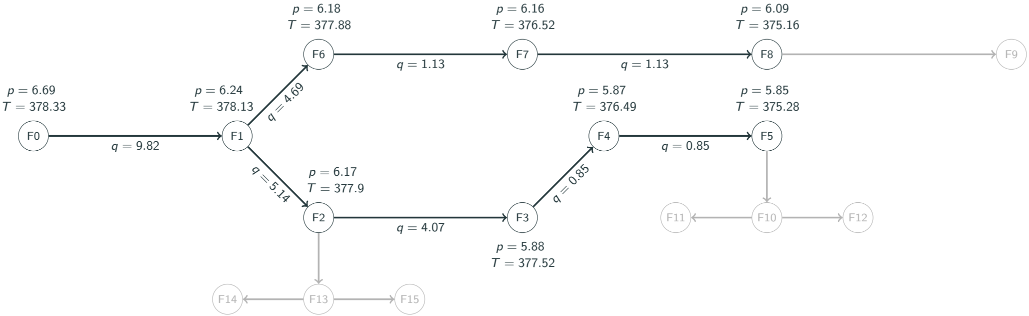
(F5,B5)	150.00
(F6,B6)	666.66
(F8,B8)	200.00
(F9,B9)	183.33
(F11,B11)	183.33
(F12,B12)	183.33
(F14,B14)	183.33
(F15,B15)	183.33

Initial Results



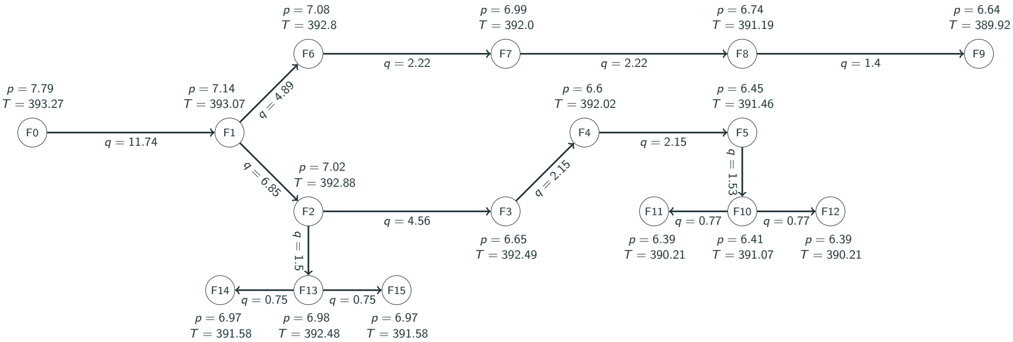
Discussion: Impact of Estimated Average Demand

If $P_a = 150\,\text{kW}$ for all $a \in A_c^c$

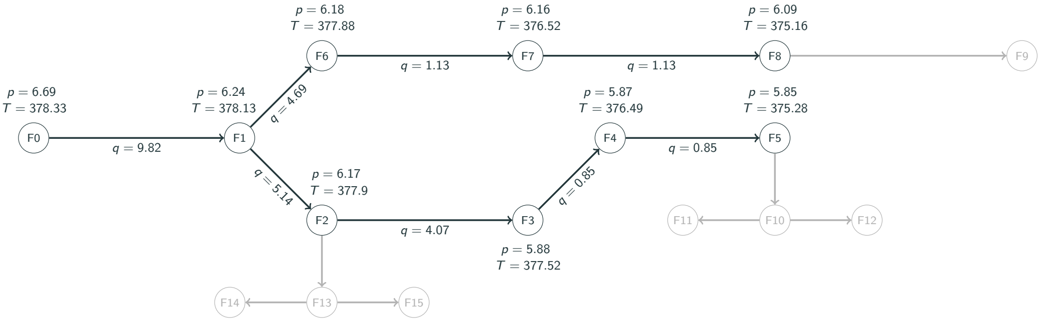


Discussion: Impact of Distance

If $P_a = 350\,\text{kW}$ for all $a \in A_c^c$



If $U_a = 0.5\,\mathrm{W\,m^{-1}\,K^{-2}} \Rightarrow 0.8\,\mathrm{W\,m^{-1}\,K^{-2}}$



Contributions of the expansion model

Conclusion

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- Nonlinear modeling of T and p behavior

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- Distance of the candidate consumer
- Pipe insulation

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mariusroland.gitlab.io